Problem 4  Shown below is the schematic of an elevator system carrying a cargo of mass $M$. Since the cable is long, the cable compliance cannot be ignored. For simplicity the cable compliance can be approximated to be constant over the length of the cable and have a stiffness $K$. An actuator of rotor inertia, $J$, winds the spool of radius, $R$, with the cable. Answer the following questions.

![Elevator system diagram](image)

Figure 1: Elevator system

a) Obtain the equations of motion.
b) Obtain the actuator torque needed for holding the cargo, $\bar{\tau}$. Obtain the transfer function from the net actuator torque $\tau_n = \tau - \bar{\tau}$ to the cargo position $y$:

$$G_1(s) = \frac{y(s)}{\tau_n(s)}.$$ 

Also obtain the transfer function from the net actuator torque to the actuator angular displacement $\theta$;

$$G_2(s) = \frac{\theta(s)}{\tau_n(s)}.$$

c) Consider feedback controls of $G_1(s)$, $G_2(s)$, as shown below. Sketch root loci of

i) Proportional Control $(k_d = 0)$ and ii) Proportional-and-Derivative Control ($k_d \neq 0$) of each of the plant transfer function, $G_1(s)$ and $G_2(s)$. Address stability and discuss the difference between the two plant transfer functions. Use any positive value for the gains and parameters involved in the systems. The results do not depend on specific values of the parameters.

Figure 2: Two feedback control systems