Problem Statement:

All ships have complex dynamics, and include turning, pitching and rolling motions. This can be forced by rudder or propulsion actions, or by disturbances from the sea in the form of waves. We are concerned here with roll motion, which is rotation about a longitudinal axis. This is illustrated in the Fig. 1

Fig. 1 –Roll motion definition

The goal of this design problem is to design a system to minimize (if not eliminate) this rolling motion when the ship is subjected to disturbances of two distinct types:

1. A tendency to roll or “heel” when turning (a known and typically constant disturbance)

2. Motion induced by surface waves of certain frequencies.

For the present study, we consider only the first or constant disturbance case.

In addition, it should be clear that we need an “actuator” of some type to create a controllable roll moment to counterbalance the natural motion of the hull.
To map out the problem, consider the word block diagram:

![Word block diagram of roll control system](image)

**Figure 2 - Word block diagram of roll control system**

This implies that the goal is to regulate the roll angle to some reference value by means of an actuator input to the ship. To begin, we can model the “Ship” block.

**Ship roll dynamics**

The roll motion of the ship can be observed simply and intuitively by looking at any object floating in water. Unless it is a perfect cylinder, it will have a stable orientation and will return to that angle if perturbed. If you are in a canoe or a dinghy or a sailboat and you rock it, it will roll side to side for several cycles and then come to rest. It is this periodic, self-righting motion that we want to model.

![Geometry of roll](image)

**Figure 3 Geometry of roll, defining center of gravity (G), center of buoyancy (B) and metacenter (M). [1]**

Figure 3 illustrates the basic forces and motions involved in roll. In normal zero roll conditions; the center of gravity (G) and the center of buoyancy (B) are vertically aligned. (B is the center of gravity of the displaced water, and the net buoyancy force acts vertically through that point.) When the hull rolls, B is displaced and now acts
along a line perpendicular to the waterline. The intersection of this line and the original vertical line between G and B defines the metacenter M.

Questions:

- Why is the ship self-righting once perturbed from zero roll?
- What lumped parameter model would approximate the behavior we can observe from experience? (Remember the video in class.)
- How would you add to that model the influence of forces or moments from turning, waves or our actuator?
- How does the design of the ship influence this model?
- How can we use knowledge of the location of G, B and M to determine parameters in our model?

A complete model of the ship roll motion will relate roll inducing torques to the resulting roll motion. Thus, we will have a basic transfer function block of the form:

\[ T_r \rightarrow G_{\text{roll}}(s) \rightarrow \theta_r \]

where \( T_r \) is the sum of all roll inducing torques and \( \theta_r \) is the resulting roll angle.

Questions:

- What are the components of \( T_r \)?
- What is a reasonable model for \( G_{\text{roll}} \)?

**Actuator**

To be able to implement the control system shown in Fig. 2 we need to impart a controllable roll torque so we can change the roll motion. To do this there are two methods in use. One shifts water between ballast tanks on either side of the ship to change the center of buoyancy location. The other uses fins on opposite sides of the hull to create differential lift with the resulting torque on the hull. We will consider this system here, since it is inherently faster and easier to implement in our control system.

The stabilizer fin method is shown pictorially in Fig. 4

The fins work by creating hydrodynamic lift, so they

![Figure 4. Illustration of lateral fins for active roll control (from [2])](image)
act very much like wings, where the cross section and angle of attack along with the stream velocity determine the net lift force.

A system produced by Sperry [2] for this purpose is shown in Fig. 5.

![The Sperry Gyrofin System and Closeup of Flow over a Fin](image)

From this illustration, it is apparent that the fins can be rotated to vary the angle of attack, $\alpha$.

From basic fluid dynamics of lift, we get the lift force equation:

$$F_L = C_L A \frac{\rho V^2}{2}$$

where $C_L$ is the lift coefficient, $A$ is the fin area (chordal) $\rho$ is the density of the fluid and $V$ is the relative stream velocity. The angle of attack enters through the lift coefficient, and the Sperry literature shows (as does much historical data) that $C_L$ varies linearly with angle of attack $\alpha$ as shown in Fig. 6.
Figure 6  Lift coefficient vs. Angle of attack for a Sperry Fin  [2]

Questions

- Assuming we can move the fin to specific angles of attack $\alpha$, what is equivalent linear relationship between angle of attack and lift force?

- How does this relationship depend upon on the speed of the ship?

If you wanted to create a block to represent the fins, it might take the form:

- What would be in the block for this transfer function?

Now consider the problem of actually moving the fins while in motion. It is basically an angular position servo, where we would command a reference angle $\alpha_r$ and get the actual rotation of the fin $\alpha$. This position control system could have a number of complexities, but for now we can assume that it is similar to a DC motor drive system we have seen in the lab. This leads to the block diagram:
Question:
- Is the position control system independent of the environment of the fin?
- If not how could you include this in the model?

**Control System Design**

Now we put all of the elements together into the original block diagram to get the model:

And now we must design the two controllers $G_c(s)$ and $G'_c(s)$

Questions:
- What are the performance requirements?
- What are the relative values of the parameters for the ship and for the actuator system?
- How can we deal with the velocity dependent gain $K_{\text{Fin}}$ right smack in the middle of the control loop?
- How do we design to deal with the disturbance $T_D$?
- How important is the steady-state error?
References

   Basics of center of buoyancy:

   Sperry – Marine Ship Roll Stabilizers

(Reference 2 will be on the class website)