Massachusetts Institute of Technology
Department of Mechanical Engineering

2.151 Advanced Systems Dynamics and Control
Assignment No. 5
Out: March 20, 2002
Due: April 5, 2002

Objectives:

1. Modelling using bond graph technique and frequency domain analysis

Problem 1: Basic characteristics of piezo actuators

When a piezoelectrical material is strained, electrical charges are generated. Due to the capacitive effect of the material a voltage is therefore induced. On the other hand, if a voltage is applied across the material, an internal stress is developed and due to the elastic nature of the material, its shape will deform (see Figure 1). In the following questions, you are asked to build a model of the material based on this description.

![Figure 1: A piece of piezoelectric material.](image)

(a) Use a two port element to model the energy transduction between the electrical domain and the mechanical domain.

(b) In the mechanical domain, the displacement (or strain) which generates the electrical charge results in elastic energy storage. On the other hand, in the electrical domain, the voltage which generates the
stress results in potential energy. Using these facts, connect the two port element of part (a) to one port elements through proper junctions. Assume that the piezoelectric material is massless and that the energy dissipation is very small in this case.

(c) Typically, the electrical-mechanical coupling effect is measured by the piezoelectric coupling coefficient \( k_{\text{piezo}} \). Assuming that all the elements in the model of part (b) are linear, derive the expressions for the coefficients \( k_1 \) and \( k_2 \), which are defined as

\[
k_1^2 = \frac{\text{electrical energy generated}}{\text{mechanical energy applied}}
\]

\[
k_2^2 = \frac{\text{mechanical energy generated}}{\text{electrical energy applied}}
\]

Also show that \( k_1 = k_2 = k_{\text{piezo}} \).

(d) Figure 2 shows two configurations for the electrodes: (i) open circuit and (ii) short circuit. Which configuration results in a stiffer material? Justify your argument.

![Figure 2: Different configurations for piezoelectric material. (a) Open circuit, (b) Closed circuit.](image)

(e) We are now interested in using the piezoelectric material to control the structural vibration of a beam. This can be accomplished by attaching the piezoelectric material to the surface of the beam as shown in Figure 3. Note that an electrical resistance has been added between the electrodes. When the beam vibrates, the vibration energy can be dissipated through the resistor. Build a simple bond graph model of the system. You can use a mass and a spring to represent the first mode of vibration of the beam.

![Figure 3: A passive vibration control system.](image)

(f) Figure 4 shows the piezoelectric stack commonly used in precision machine design. This piezoelectric transduction element is made of a stack of piezoelectric discs. By applying the same voltage to each disc, each disc elongates and consequently the total displacement is the sum of all the disc displacements. This design is used to increase the range of motion. Build a simple bond graph model of the system.
Problem 2: Transfer functions

Figure 5 shows the experimental setup used to conduct frequency response tests for the magnetic bearing system. During the tests, the rotor was either at rest or spinning using a controller to stabilize the system. Since the system is originally unstable, these tests can be conducted after the system is stabilized. A HP dynamic signal analyzer is used to send a swept sine signal which acts as a disturbance to one of the bearings. The signal frequency ranges from 0.1 Hz to 10 kHz.

![Diagram of the experimental setup](image)

(a) Assuming that $\alpha = 31.250$, $\beta = 25$ and a PD controller $(k_p + k_d s)$ is used with parameters $k_p = 17041.40$ and $k_d = 35.537$. Referring to Figure 6, compute the forward loop transfer function $G(s)G_c(s)$, the return difference transfer function $1 + G(s)G_c(s)$, the sensitivity transfer function $[1 + G(s)G_c(s)]^{-1}$, and the closed-loop transfer function $[1 + G(s)G_c(s)]^{-1} G(s)G_c(s)$.

(b) Prepare the Bode plots of the corresponding transfer functions found in part (a) and comment on your results.

(c) Determine the system bandwidth $f_b$.

(d) Assume that the system noise $N(s)$ can be either $N(s) = N_0 \sin(\pi f dt)$ or $N(s) = N_0 \sin(40\pi f dt)$ where $N_0 = 5 \mu m$. For each case, plot the time response of the system output as a function of time. Comment on your results.

(e) Compute the zero, $z_0$, introduced by the PD controller. Determine its effect on the time response by simulating two new systems with zeros of magnitude $|z_1| = 10|z_0|$ and $|z_2| = 0.1|z_0|$ (by changing the value of $k_p$). Perform the simulation using a step reference of $200 \mu m$. Comment on your results.
Figure 6: System block diagram.