Problem Set 1

 Assigned: Sept. 15, 2004  
 Due: Sept. 27, 2004

Problem 1: A locomotive pulling a single car on a straight, flat track is sketched below. The locomotive may be considered to generate a prescribed force $F(t)$. The coupling between the locomotive and the car has both stiffness and damping properties. The car is subject to both rolling friction at the wheels and aerodynamic drag.

(a) Make a sketch of the system, identifying the major system elements and a velocity reference direction. Identify the energy storage elements in your system. Make a suitable model of the coupler. Do you think that the coupler elements should be in series or in parallel?

(b) Construct the system linear graph.

Problem 2: A four story parking building, shown below, is located in an earthquake zone. Model each floor as a lumped mass element, connected by steel girders to the floors above and below. Assume that during an earthquake the ground moves horizontally with a specified velocity, and that the girder structure has a finite translational stiffness, so that a sideways displacement on a floor results in a restoring force proportional to the displacement. Generate a linear graph model of the parking building. What would be the order of a set of state equations describing your model? Identify the state variables that would result from the model.
**Problem 3:** A parts assembly station on a production line exhibits a severe vibration problem. A simplified schematic representation is shown below. Two large tables of mass $m_1$ and $m_2$ are each mounted to a sliding metal plate on resilient rubber mounts, with shear stiffness $K_1$ and $K_2$ as shown. The tables are each subjected to a vibrational excitation force, $F_1(t)$ and $F_2(t)$. The plates are able to slide with viscous friction a second pair of deformable rubber mounts, with shear stiffnesses $K_3$ and $K_4$. The viscous sliding coefficients are $B_1$ and $B_2$. The two plates are coupled by a shaft with longitudinal stiffness $K_5$. Draw a linear graph for the system using the two forces $F_1$ and $F_2$ as inputs.

**Problem 4:** A one-quarter car model is useful for studying the effects of the road surface on the vertical motion of a car body. The model includes the tire stiffness as well as the suspension damping and stiffness, and supports a mass of one quarter of that of the car, as shown below. The roadway is modeled as providing a vertical velocity input to the tire as the car travels along the road. (Because the gravitational force is constant, it may be omitted when modeling incremental motions about an equilibrium point.)

(a) Construct the system linear graph.
(b) Identify the system state variables.
(c) Derive the state equations and express them in matrix form.
(d) Derive output equations for the total force acting on the car body mass from the suspension spring and damper.

**Problem 5:** Seismometers are used to measure the motion of the earth’s surface. A schematic drawing of a simple seismometer is shown below. A proof mass is suspended in springs and slides horizontally on a viscous friction material. The relative displacement of the proof mass with respect to the instrument case is used as a measure of the severity of an earthquake.

(a) Construct a linear graph model of the system.
(b) How many independent energy storage elements are there? What are the system state variables?
(c) Derive the system state equations and express them in matrix form.

(d) Derive an output equation for the instrument reading, that is the relative displacement of the proof mass with respect to the instrument case.

**Problem 6:** L-C circuits are commonly used as tuning circuits in radio receivers. In the schematics below the remote radio transmitter, the propagation path, and antenna are modeled as a Thévenin source with source resistance \( R_s \). The circuits consist of an inductor \( L \) (with a small but finite resistance \( R_L \)) and a variable capacitor \( C \) connected in series or parallel as shown. The output voltage \( v_o \) is amplified and demodulated into an audio signal. For the series L-C circuit in (a), and the parallel L-C circuit in (b):

(a) Draw the linear graph and normal tree for the tuning circuit.

(b) Derive a set of state equations for the circuit.

(c) Develop an output equation for the voltage \( v_o(t) \).