**Problem 1:**

(a) The system may be modeled at many levels of complexity. A simple model is shown in the sketch below:

The energy storage elements are (1) the mass of the locomotive $m_1$, (2) the mass of the car $m_2$, and the compliance (spring) $K$ in the coupling. The various drag forces are accounted for by the viscous friction elements $B_1$ and $B_2$.

The coupler elements should be in parallel. Think of the implications if they were in series.

(b) The system linear graph is shown below.

**Problem 2:** The basic linear graph:

A practical system will have energy dissipation. You can argue how you would include dissipative elements in the linear graph.
**Problem 3:** The linear graph is:

![Linear Graph](image)

**Problem 4:**
(a) The linear graph is:

![Linear Graph](image)

(b) The state variables will be (1) the velocity of the car mass $v_{mc}$, (2) the velocity of the wheel/axle mass $v_{mw}$, (3) the force in the suspension stiffness $F_{ks}$, and (4) the force in the tire stiffness $F_{kt}$.

(c) The state equations are:

$$
\begin{bmatrix}
\dot{v}_{mc} \\
\dot{v}_{mw} \\
\dot{F}_{ks} \\
\dot{F}_{kt}
\end{bmatrix}
\begin{bmatrix}
-B_s/m_c & B_s/m_c & 1/m_c & 0 \\
B_s/m_w & -B_s/m_w & -1/m_w & 1/m_w \\
-K_s & K_s & 0 & 0 \\
0 & -K_t & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v_{mc} \\
v_{mw} \\
F_{ks} \\
F_{kt}
\end{bmatrix}
= 
0
$$

(d) The output equation is

$$y = 
\begin{bmatrix}
-B_s & B_s & 1 & 0
\end{bmatrix}
\begin{bmatrix}
v_{mc} \\
v_{mw} \\
F_{ks} \\
F_{kt}
\end{bmatrix}
+ [0] V_s
$$

**Problem 5:** The two springs are not independent, they share a common velocity, and the equivalent stiffness is $K_{eq} = 2K$.

(a) The linear graph is:

![Linear Graph](image)
(b) There are two independent energy storage elements. The state variables are (1) the velocity of the proof mass \( v_m \), and the force in the equivalent spring \( F_{K_{eq}} \).

(c) The state equations are

\[
\begin{bmatrix}
\dot{v}_m \\
\dot{F}_{K_{eq}}
\end{bmatrix} = \begin{bmatrix}
-B/m & 1/m \\
-K_{eq} & 0
\end{bmatrix} \begin{bmatrix}
v_m \\
F_{K_{eq}}
\end{bmatrix} + \begin{bmatrix}
B/m \\
K_{eq}
\end{bmatrix} V_c
\]

(d) The output equation is:

\[
y = \begin{bmatrix}
0 \\
1/K_{eq}
\end{bmatrix} \begin{bmatrix}
v_m \\
F_{K_{eq}}
\end{bmatrix} + [0] V_c
\]

**Problem 6:** For the series tuning circuit (System (a)):

(a) The linear graph and normal tree are:

(b) The state equations are:

\[
\begin{bmatrix}
\dot{v}_c \\
\dot{i}_L
\end{bmatrix} = \begin{bmatrix}
0 & 1/C \\
-1/L & -(R_s + R_L)/L
\end{bmatrix} \begin{bmatrix}
v_c \\
i_L
\end{bmatrix} + \begin{bmatrix}
0 \\
1/L
\end{bmatrix} V_s
\]

(c) The output equations is

\[
y = \begin{bmatrix}
0 \\
-R_s
\end{bmatrix} \begin{bmatrix}
v_c \\
i_L
\end{bmatrix} + [1] V_s
\]

For the parallel tuning circuit (System (b)):

(a) The linear graph and normal tree are:

(b) The state equations are:

\[
\begin{bmatrix}
\dot{v}_c \\
\dot{i}_L
\end{bmatrix} = \begin{bmatrix}
-1/C R_s & -1/C \\
1/L & -R_L/L
\end{bmatrix} \begin{bmatrix}
v_c \\
i_L
\end{bmatrix} + \begin{bmatrix}
1/C R_s \\
0
\end{bmatrix} V_s
\]
(e) The output equation is

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + [0] V_s \]