Problem Set 5

Assigned: Nov. 1, 2004
Due: Nov. 8, 2004

Problem 1: Consider the electrical circuit shown below:

The output is $v(t)$ as shown.

(a) Determine the system controllability matrix.

(b) Find a relationship between $R, L, C$ that renders the system uncontrollable.

(c) Determine the transfer function relating $V(s)$ to $I(s)$ when the system is uncontrollable.

(d) Is the system observable when it is uncontrollable.

Problem 2: Consider the “T” electrical filter shown below:

The linear graph modeling method produces the following state equations

\[
\begin{bmatrix}
\dot{v}_{C_1} \\
\dot{v}_{C_2} \\
\dot{v}_{C_3} \\
i_L \\
\end{bmatrix}
= \begin{bmatrix}
-1/RC_1 & -1/RC_1 & 0 & 1/C_1 & 0 \\
-1/RC_2 & -1/RC_2 & 0 & 0 & 0 \\
0 & 0 & 1/C_3 & 0 & 0 \\
-1/L & 0 & -1/L & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
v_{C_1} \\
v_{C_2} \\
v_{C_3} \\
i_L \\
\end{bmatrix}
+ \begin{bmatrix}
1/RC_1 \\
0 \\
0 \\
1/L \\
\end{bmatrix} V(t)
\]

and transfer function

\[
\frac{V_o(s)}{V(s)} = \frac{RC_1 C_2 C_3 L s^3 + C_1 C_3 L s^2}{RC_1 C_2 C_3 L s^3 + LC_3 (C_1 + C_2) s^2 + RC_2 (C_1 + C_3) s + (C_1 + C_2 + C_3)}
\]
(a) Without computing the controllability matrix, determine whether the system is controllable. Explain your answer.

(b) Reduce the system to a third-order system, and show the transfer function is the same as shown above.

(c) Do you expect the reduced system to be controllable.

**Problem 3:** A system with two inputs and two outputs is described by a pair of second-order coupled equations:

\[
\ddot{y}_1 + 3\dot{y}_1 + 2y_2 = u_1 + 2u_2 + 2\dot{u}_2 \\
\ddot{y}_2 + 4\dot{y}_1 + 3y_2 = \ddot{u}_2 + 3\dot{u}_2 + u_1
\]

Derive a set of state equations for this system, derive the eigenvalues and comment on the system stability.

**Problem 4:** By considering the time derivative of the energy stored in the system, use Lyapunov concepts to determine the stability of the electrical system:

![Electrical circuit diagram]

**Problem 5:** Repeat Problem 4 for the mechanical system shown below - where the frictional force is nonlinear, \( F_B = -B\text{sgn}\{v_m\} \).

![Mechanical system diagram]

**Problem 6:** The following system is specified in Jordan canonical form

\[
\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u,
\]

Determine the stability of the system – is it asymptotically stable, unstable or neutrally stable?