Problem Set 6 Solutions

Problem 1  There are several approaches to this problem. For example, a transfer function representation is always controllable and observable, in fact the transfer function represents the controllable and observable subsystem. Thus we have implicitly assumed controllability in writing the transfer function in step (3).

Further - look at the transfer function when \( \alpha_1 = \alpha_2 \)

\[
H(s) = \frac{(c_1 + c_2)(s - \alpha)}{s^2 - 2\alpha s + \alpha^2} = \frac{c_1 + c_2}{s - \alpha}
\]

so that pole/zero cancellation has occurred - the system is uncontrollable. Under such conditions, step(4) is invalid!

Problem 2  Using Matlab’s function \texttt{place()}, we find \( K \) to be:

(a)  \[
K = \begin{bmatrix} 0 & 0 & 6 \\ 2 & 3 & 0 \end{bmatrix}
\]

or

\[
K = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 1 & 0 \end{bmatrix}
\]

(b)  \[
K = \begin{bmatrix} 0 & 0 & 7 \\ 1 & 0 & 0 \end{bmatrix}
\]

Problem 3

(a)  To obtain \( \theta_2 \), the output matrix must be

\[
C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}
\]

\[
\theta_2(s) = k \frac{s^3J_1J_2 + s^2(J_1b_2 + J_2b_1) + \frac{kb_2 + kJ_2}{s + k}T_m(s)}{J_1s^2 + b_1s + k} + \frac{kb_2 + kJ_1}{s + k}T_d(s)
\]

(b)  From the diagram we see that the desired closed loop poles are \(-2 \cos(22.5^\circ) \pm j2 \sin(22.5^\circ) = 1.8478 \pm 0.7654j\) and \(-2 \cos(67.5^\circ) \pm j2 \sin(67.5^\circ) = 0.7654 \pm 1.8478j\). Then using Matlab’s \texttt{place()} function

\[
K = \begin{bmatrix} -131.6 & 6.16 & 131.6 & -10.9 \\ 174.1 & 5.37 & -145.9 & 8.05 \end{bmatrix}
\]
or

\[ K = \begin{bmatrix} -131.7 & 6.17 & 188.2 & 10.79 \\ 145.8 & -5.43 & -145.9 & 8.05 \end{bmatrix} \]

**Problem 4**  The system transfer function is

\[ H(s) = C(sI - A)^{-1}B = \frac{2}{s^2 + 3s + 2} \]

which is a unity gain Type 0 system. To form the integral controller, augment the system with an extra state equation representing the integration of the output

\[ \dot{x}_3 = x_1 \]

and form the state feedback matrix \( K \) and the control

\[ u = -Kx \]

The augmented homogeneous system is described by

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ -2 & -3 & 0 \\ 1 & 0 & 0 \end{bmatrix} x \\
y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x
\end{align*}
\]

The Matlab function `acker()` is used to find the gains for closed-loop eignenvalues of \((-1, -1, -2)\), giving

\[ K = \begin{bmatrix} 1.5 & 0.5 & 1 \end{bmatrix} \]

For integral control with a non-zero set-point we have

\[ \dot{x}_3 = x_1 - y_{ref} \]

giving the closed-loop state-equations, based on \( A - BK \):

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ -5 & -4 & -2 \\ 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} y_{ref} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} w \\
y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x
\end{align*}
\]

and Matlab’s `step()` function was used to generate the following step-responses to \( y_{ref} \) and \( w \).
Command and Disturbance Step Response

Time (sec)

Step Response

Command

Disturbance