Holography

- Preamble: modulation and demodulation
- The principle of wavefront reconstruction
- The Leith-Upatnieks hologram
- The Gabor hologram
- Image locations and magnification
- Holography of three-dimensional scenes
- Transmission and reflection holograms
- Rainbow hologram
Modulation & Demodulation

- Principle borrowed from radio telecommunications
- Idea is to take baseband signal (e.g. speech, music, with maximum frequencies up to ~20kHz) and modulate it onto a carrier signal which is a simple tone at the frequency where the radio station emits, e.g. 104.3 MHz (that’s Boston’s WBCN station)
- One of the benefits of modulation is that radio stations can be multiplexed by using a different emission frequency each
- After selecting the desired station, the receiver follows a process of demodulation which recovers the baseband signal and sends it to the speakers.
Types of modulation

- Amplitude modulation (AM)
- Frequency modulation (FM)
- Phase modulation (PM)
- Digital methods (Amplitude Shift Keying – ASK, Frequency Shift Keying – FSK, Phase Shift Keying – PSK, etc.)

used in radio at low frequencies only (“AM band” = 535kHz to 1.7MHz); as we will see, it is an almost-exact analog of holography.

dominant in commercial radio (“FM band” = 88MHz to 108MHz); there is an analog in optics, called “spectral holography,” but it is beyond the scope of the class.
Amplitude modulation

\[ f(x) \]  
(baseband)

modulated \[ f(x) = f(x) \times \cos(2\pi u_c x) \]

\( u_c \): carrier frequency
AM in the frequency domain

spectrum of $f(x)$

spectrum of modulated $f(x)$
AM in the frequency domain

spectrum of $f(x)$

(zoom-in)

spectrum of modulated $f(x)$

(zoom-in)
AM in the frequency domain

\[ f(x) \rightarrow F(u) \]

\[ f(x)\cos(2\pi u_c x) = f(x) \times \frac{1}{2} [e^{i 2\pi u_c x} + e^{-i 2\pi u_c x}] \]

\[ \rightarrow F(u) * \frac{1}{2} [\delta(u - u_c) + \delta(u - u_c)] = \]

\[ = \frac{1}{2} [F(u - u_c) + F(u - u_c)] \]

modulation in the space domain

modulation in the frequency domain: two replicas of the baseband spectrum, centered on the carrier frequency
Modulation

\[ f(x) \times \cos(2\pi u_c x) \rightarrow \text{modulated } f(x) \]
Demodulation

modulated $f(x)$ \times \cos(2\pi f_c x) \rightarrow \text{low-pass filter} \rightarrow f(x)$

- simple carrier tone
- must accommodate baseband spectrum
Demodulation

\[ f(x) \times \cos^2(2\pi u_x x) \]

spectrum of \( f(x) \times \cos^2(2\pi u_x x) \)
Demodulation

\[ f(x) \times \cos^2(2\pi u_c x) \]

spectrum of \( f(x) \times \cos^2(2\pi u_c x) \)

LP filter pass-band
The wavefront reconstruction problem

- Wavefront is the amplitude (i.e. magnitude and phase) of the electric field as function of position
- Traditional coherent imaging results in intensity images (because detectors do not respond fast enough at optical frequencies) → magnitude information is recovered but phase information is lost
- Can we imprint *intensity* information on an optical wave? YES → *photography* (known since the 1840’s)
- Can we imprint *wavefront* information on an optical wave? YES → *holography* (Gabor, late 1940s)
Photography: recording

incident illumination
(laser beam or white light)

imaging system

film records intensity information

\[ |S|^2 \]
Photography: reconstructing the intensity

incident illumination (laser beam or white light)

\[ |S|^2 \]

imaging system

at the image plane, an intensity pattern is formed that replicates the originally recorded intensity
Holography: recording

incident illumination
(laser beam*)

imaging system

film records the interference pattern
(interferogram) of the object wavefront
and the reference wavefront

reference beam
(split from the same laser)

\[ |R + S|^2 \]

*in general, the illumination must be quasi-monochromatic,
and spatially mutually coherent
with the reference beam throughout the wavefront
Holography: reconstructing the wavefront

illumination: replicates the reference beam

\[ |R + S|^2 \]

what is the field at the image plane?
Holography: reconstructing the wavefront

The field being imaged is:

\[ R \times |R + S|^2 = R \times (|R|^2 + |S|^2 + R^*S + R S^*) = \]

\[ = R \times (|R|^2 + |S|^2) + |R|^2 S + R^2 S^* \]
Holography: reconstructing the wavefront

\[ |R + S|^2 \]

take the simplest possible reference wave, a **plane wave**:

\[ R = e^{i2 \pi u_0 x} \]

spatial frequency \( u_0 = \frac{\sin \theta_0}{\lambda} \)

then the reconstructed field is:

\[ = e^{i2 \pi u_0 x} \times \left(1 + |S|^2\right) + S + e^{i2 \pi (2u_0) x} S^* \]

1. \( e^{i2 \pi u_0 x} \times \left(1 + |S|^2\right) \)
2. \( S \)
3. \( e^{i2 \pi (2u_0) x} S^* \)
Holography: reconstructing the wavefront

\[ \text{fields departing from the hologram} \]

1. \[ e^{i2\pi u_0 x}(1 + |S|^2) \rightarrow \theta_0 \]
2. \[ S \rightarrow \text{on-axis} \]
3. \[ e^{i2\pi u_0 x}S^* \rightarrow 2\theta_0 \]
Holography: reconstructing the wavefront

\[ |R + S|^2 \]

\[ R \]

\[ S \rightarrow \text{on-axis} \]

\[ e^{i2\pi(2u_0)x} S^* \rightarrow 2\theta_0 \]

\[ e^{i2\pi u_0 x} \times (1 + |S|^2) \rightarrow \theta_0 \]

\[ \text{fields departing from the hologram} \]
Filtering the wavefront: bandlimited signal

\[ S \text{ has bandwidth } w, \text{ i.e. } \mathcal{F}\{S\} \neq 0 \text{ within circle of radius } w \]
Filtering the wavefront: bandlimited signal

Term $\textbf{1} \rightarrow 1 + |S|^2$ has bandwidth $2w$, because

$$\mathcal{F}\{ |S|^2 \} = \mathcal{F}\{S\} \ast \mathcal{F}\{S^*\} = \mathcal{F}\{S\} \otimes \mathcal{F}\{S\}$$
Filtering the wavefront:
Fourier transform description

\[ u \]

\[ v \]

\[ w \]

\[ 2w \]

\[ u_0 \]

\[ 2u_0 \]

\[ 2 \]

\[ 1 \]

\[ 3 \]
Filtering the wavefront: Fourier transform description

1. Original spectrum
2. Autocorrelation of the original spectrum
3. Original spectrum but phase-conjugated: inside-out, or “pseudo-scopic”
Filtering the wavefront: Fourier transform description

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Filtering the wavefront: Fourier transform description

A low-pass filter of passband $w$ or slightly greater permits the desired term $\mathbf{2}$ to pass, and eliminates the undesirable terms $\mathbf{1}$ and $\mathbf{3}$. 
Holography: reconstructing the wavefront

- **illuminat**ion: replicates the reference beam
- **hologram**: $|R + S|^2$

4F system with Fourier plane filter

the field at the image plane replicates the original $S$ stored in the hologram
Filtering the wavefront: Fourier transform description

Potential problem: spectra overlap!
Filtering the wavefront: Fourier transform description

Spectra should not overlap, i.e. $u_0 - w > \frac{w}{2} \iff u_0 > \frac{3}{2}w$
Leith-Upatnieks vs Gabor hologram

Leith-Upatnieks

\[ |R + S|^2 \]

\[ R = e^{i2\pi \mu_0 x} \]

Gabor

\[ |R + S|^2 \]

\[ R = 1 \]
Analogy between the Leith-Upatnieks hologram and amplitude modulation (AM)

**AM Radio**
- Modulation
  - $f(x)$
  - $\cos(2\pi u_0 x)$
  - Modulated $f(x)$

**Holography**
- Recording
  - $S$
  - $R = e^{i2\pi u_0 x}$
  - $|R + S|^2$

**Demodulation**
- Modulated $f(x)$
- $\cos(2\pi u_0 x)$
- Low-pass filter
- $f(x)$

**Reconstruction**
- $|R + S|^2$
- $R = e^{i2\pi u_0 x}$
- Low-pass filter
- $S$
Image locations and magnification

Reference source
$x_r, y_r, z_r$

Object source
$x_0, y_0, z_0$

Recording medium

Reconstruction source
$x_p, y_p, z_p$

Hologram
Image locations and magnification

\[ z_i = \left( \frac{1}{z_p} \pm \frac{\lambda_2}{\lambda_1 z_r} \pm \frac{\lambda_2}{\lambda_1 z_0} \right)^{-1} \]

\[ x_i = \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} x_0 \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} x_r + \frac{z_i}{z_p} x_p \]

\[ y_i = \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} y_0 \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} y_r + \frac{z_i}{z_p} y_p \]

Transverse Magnification

\[ M_t = \left| \frac{\partial x_i}{\partial x_0} \right| = \left| \frac{\partial y_i}{\partial y_0} \right| = \left| \frac{\lambda_2 z_i}{\lambda_1 z_0} \right| = \left| 1 - \frac{z_0}{z_r} \mp \frac{\lambda_1 z_0}{\lambda_2 z_p} \right|^{-1} \]

Axial Magnification

\[ M_a = \left| \frac{\partial z_i}{\partial z_0} \right| = \left| \frac{\partial}{\partial z_0} \left( \frac{1}{z_p} \pm \frac{\lambda_2}{\lambda_1 z_r} \pm \frac{\lambda_2}{\lambda_1 z_0} \right) \right|^{-1} = \frac{\lambda_1}{\lambda_2} M_t^2 \]
Holography of Three-Dimensional Scenes
Orthoscopic and Pseudoscopic

(a) Hologram recording

(b) Image reconstruction

Virtual image (orthoscopic)

Collimated reference beam

Photographic plate

Object illumination

Hologram

Real Image (Pseudoscopic)
Holography of Three-Dimensional Scenes

(a)

(b)
Transmission and Reflection Holograms

- Reflection hologram

(a) Recording plane

(b) Observer

Hologram
Transmission and Reflection Holograms
Rainbow hologram (Record)
Rainbow hologram (Reconstruct)