This quiz is closed book, except for the quiz announcement handout and one additional page of notes that you may have prepared. The mystery circuit from the handout is repeated, for your convenience, at the end of this quiz. The circuit variables referred to in the quiz are indicated in this diagram.

Please write clear and concise answers to the questions in the spaces provided in this booklet. You may use extra paper, if needed, but the spaces we provide are surely big enough to contain the simple answers we are looking for. You must be brief but complete and clear—Lem has enough problems to worry about these days without having to dig your pearls of wisdom out of a rubble of disorganized thoughts. We will certainly give you no credit for an answer that Prof. Logue cannot decode.

You must put your name in the place provided at the top of each page of the quiz. You must also put your name, the name of your tutor, and the name of your recitation instructor in the places provided below:

Name: SOLUTIONS
Tutor: 
Recitation Instructor: 

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1. The circuit shown above is a subcircuit of the mystery amplifier. The subcircuit consists of the two power-supply sources (drawn as batteries), the resistors $R_1$ and $R_2$, and the incremental input source, with strength $v_i$. Determine the Thévenin equivalent circuit for this subcircuit. That is, give the open-circuit voltage and the equivalent resistance as seen from the indicated set of terminals. Show your calculations in the space provided below. (10 pts.)

**$R_{th}$:** turn off all independent sources.

$$R_{th} = R_i \parallel R_2 = \frac{R_i R_2}{R_i + R_2}$$

**$V_{oc}$:** since $R_i = R_2$, voltage at $A$ is 0 V by voltage divider and symmetry of circuit.

=>$V_{oc} = v_i$
2. (a) In the mystery amplifier, what is the voltage, \( v_3 \), on the gate of \( Q_3 \), relative to the indicated ground? Write your answer, in terms of the unknown resistance, \( R_4 \), in the space provided below. (10 pts.)

\[
V_3 \text{ is } 15\text{V} \text{ minus voltage drop across } R_3. \\
\text{current through } R_3, \ i_{R3} = \frac{15-(-15)}{R_3+R_4} = \frac{30}{R_3+R_4}
\]

\[=> V_3 = 15 - \frac{30}{R_3+R_4} R_3 \]

(b) Assuming that the potential, \( e_3 \), on the drain of \( Q_3 \) is high enough so that \( Q_3 \) is operating in the saturated region, what is the current, \( i_3 \), flowing into its drain terminal? Write your answer, in terms of the unknown resistance, \( R_4 \), in the space provided below. (10 pts.)

Assuming \( Q_3 \) in saturation, \( i_3 = \frac{K}{2} (V_{es3} - V_T)^2 \)

\[V_{es3} = V_3 - (-15) = V_3 + 15\]

\[= 15 - \frac{30}{R_3 + R_4} + 15\]

\[= 30 - \frac{30R_3}{R_3 + R_4}\]

\[=> i_3 = \frac{K}{2} \left( 30 - \frac{30R_3}{R_3 + R_4} - 1 \right)^2 \]

\[i_3 = .001 \left( 29 - \frac{30(47,000)}{47,000 + R_4} \right)^2 \]
3. (a) In one or two concise sentences explain why the bias currents, \( i_1 \) and \( i_2 \), in the transistors \( Q_1 \) and \( Q_2 \) are equal. Write your explanation in the space provided below. (10 pts.)

Both \( Q_1 \) and \( Q_2 \) have the same source voltage, \( E_s \).
\( Q_1 \) and \( Q_2 \) both have gate voltages of 0V
\[ V_{GSS1} = V_{GSS2} \implies I_1 = I_2 \]

(b) Lem has measured the output voltage, \( V_O \), on an installed unit. He has determined that the bias voltage, \( V_O \), at the output is 10 volts, relative to the indicated ground. What is the bias potential, \( E_s \), on the node where the sources of \( Q_1 \) and \( Q_2 \) join? Argue that the bias potential that you deduced justifies the assumption that \( Q_3 \) is saturated. Write your answers in the space provided below. (10 pts.)

\[ V_O = 15 - 2R_5 = 10 \implies i_2 = \frac{5}{k} = 0.5 \text{ mA} \]

\[ i_2 = \frac{k}{2}(V_{GSS2} - V_T)^2 \]

\[ 0.5 \text{ mA} = \frac{k}{2}(0 - E_s - V_T)^2 \implies E_s = -1.701 \text{ V} \]

\[ V_{DSS} = E_s - (-15) = 13.293 \text{ V} \]

since \( I_3 = 2I_2 = 1 \text{ mA} = \frac{k}{2}(V_{GSS3} - V_T)^2 \implies V_{GSS3} = 2 \text{ V} \]

\[ V_{GSS3} \geq V_T \checkmark \]

\[ V_{DSS} > V_{GSS3} - V_T \checkmark \implies Q_3 \text{ in saturation} \]

(c) What is the value of \( R_4 \) that will give the output bias voltage, \( V_O = 10 \text{ volts} \), as in part 3b? Write your answers in the space provided below. (10 pts.)

from 3a) \( i_1 = i_2 \), \( i_3 = i_1 + i_2 = 1 \text{ mA} \)

from 2b) \( i_3 = \frac{k}{2}(29 - \frac{30R_3}{R_3 + R_4})^2 = 1 \text{ mA} \)

\[ K = 2 \text{ mA/V}^2 \]

\[ R_3 = 47 \text{ K} \]

\[ \implies R_4 \approx 3.357 \text{ K} \]
4. Assume, as postulated above, that the output bias is 10 volts.

(a) What is the incremental change in drain current, $i_2$, in $Q_2$ as a function of an incremental change in the gate voltage, $v_1$, of $Q_1$? Write your answer in the space provided below. (15 pts.)

Because $i_1 + i_2 = i_3 \Rightarrow \Delta i_1 = -\Delta i_2$

\[
\begin{align*}
\gamma_i &= K \left( V_{GS1} - V_T \right) \cdot V_{GS1} \\
\frac{i_1}{i_2} &= K \frac{0 - E_x - V_T}{V_{GS1}} \\
&= 0.707 \times 10^{-3} \gamma_i \\
\Rightarrow \quad i_2 &= -i_1 = -0.707 \times 10^{-3} \gamma_i
\end{align*}
\]

(b) What is the voltage gain of this circuit? That is, what is the ratio of the incremental change in the output voltage, $v_o$, to the incremental change of the gate voltage, $v_1$, of $Q_1$? Write your answer in the space provided below. (10 pts.)

\[
\begin{align*}
v_o &= -i_2 R_s \\
v_o &= 0.707 \times 10^{-3} R_s \gamma_i \\
\frac{v_o}{v_i} &= 7.07 \\
R_s &= 10 K
\end{align*}
\]

(c) For what voltage $v_1$ on the gate of $Q_1$ does $Q_2$ become cutoff? (A MOS transistor is said to be cutoff if its drain current is zero. Write your answer in the space provided below. (15 pts.)

$Q_2$ becomes cutoff when $V_{GS2} = V_T = 1V$

$V_{GS2} = 0 - E_x = 1V$

$\Rightarrow E_x = -1V$

When $i_2 = 0$, $i_1 = i_3 = 1mA$

\[
\begin{align*}
i_1 &= \frac{K}{2} \left( V_{GS1} - V_T \right)^2 \\
\therefore 1mA &= \frac{2mA/N^2 \left( V_1 - E_x - V_T \right)^2}{2} \\
\Rightarrow V_1 &= 1V
\end{align*}
\]
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All transistors have $V_t = 1$ volt and $K = 2$ mA/volt$^2$.

$R_1 = 100k$  $R_2 = 100k$  $R_3 = 47k$  $R_4 = ???$  $R_5 = 10k$