MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

6.002 Circuits and Electronics Quiz #2
November 10, 2004

YOUR NAME

SOLUTIONS

Recitation Instructor / TA

General Instructions:

1. Please verify that there are 18 pages in your exam booklet.

2. Please do all of your work in the spaces provided in this examination booklet. In particular, try to do your work for each question within the boundaries of the question, or on the back side of the page preceding the question. Extra pages are also available at the end of the booklet. Place the answer to each question within the appropriate answer box.

3. You may use two double-sided pages of notes and a calculator while taking this exam.

For examiner's use only:

<table>
<thead>
<tr>
<th>Part</th>
<th>1</th>
<th>2A</th>
<th>2B</th>
<th>2C</th>
<th>2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part</th>
<th>3A</th>
<th>3B</th>
<th>4</th>
<th>5A</th>
<th>5B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TOTAL SCORE
Problem 1  (12 Points)

Find a Thévenin equivalent model of the circuit shown in Figure 1 as viewed from the terminal pair at the right of the circuit. (That is, find the Thévenin voltage and resistance $V_{TH}, R_{TH}$ that characterize the system.) You may ignore the degenerate case that occurs when $\alpha = -(R_1+R_2)$.

\[ KCL @ \text{Top of } R_1: \quad I = \frac{V_{TH}}{R_2} + I_0 \]

\[ V_{TH} = (\alpha + R_1)I = (\alpha + R_1)\frac{V_{TH}}{R_2} - \frac{(\alpha + R_2)\frac{V_{TH}}{R_2} + (\alpha + R_2)I_0}{R_2} \]

\[ V_{TH} = (1 + \frac{\alpha + R_1}{R_2}) = (\alpha + R_1)(\frac{V_{TH}}{R_2} + I_0) \]

\[ V_{TH} = \frac{(\alpha + R_1)R_2}{R_2 + R_1 + R_2} (\frac{V_{TH}}{R_2} + I_0) \]

\[ V_{TH} = \frac{(\alpha + R_1)R_2}{R_2 + R_1 + R_2} I_0 \]

\[ R_{TH} = \frac{V_{TH}}{\frac{V_{TH}}{R_2} | I_0 = 0} = \frac{(\alpha + R_1)R_2}{R_2 + R_1 + R_2} \]
This problem concerns the MOSFET amplifier shown in Figure 1.

Figure 1

Find the value of the dc input voltage \( V_i \) such that the dc output voltage \( V_{OUT} = 5 \) V. Assume that the MOSFET operates in the saturation region \( i_d = \frac{1}{2} k (V_G - V_T)^2 \), and that parameters \( k = 2 \) mA/V² and \( V_T = 1 \) V.

\[
10 - \frac{10 - V_o}{6 \times 10^{-3}} = 10 \Rightarrow \frac{V_G - V_T}{2} = \frac{(V_G - V_T)^2}{6 \times 10^{-3}}
\]

\[V_G = V_T + 1 \Rightarrow V_G = 2 \text{V}, V_T = 1 \text{V}, V_G > V_T = 1 \text{V}
\]

\[V_{OUT} + 1 \frac{k (V_G - V_T)}{2} = V_i = 2 \text{V} \Rightarrow K_{OUT}
\]

\[
V_{TH} = \frac{(\alpha + R)R_2 I_D}{\alpha + R + R_2}
\]

\[
R_{TH} = \frac{(\alpha + R)R_2}{\alpha + R + R_2}
\]
Problem 2  (25 Points)

This problem concerns the MOSFET amplifier shown in Figure 2.

![MOSFET Amplifier Diagram](image)

Figure 2

(2A) Find the value of the dc input voltage $V_i$ such that the dc output voltage $V_{\text{OUT}} = 5$ V. Assume that the MOSFET operates in the saturation region $i_D = \frac{1}{2}k(v_{GS} - V_T)^3$, and has parameters $k = 2 \text{ mA/V}^2$ and $V_T = 1$ V.

$$i_D = \frac{10 - V_o}{5 \times 10^3} = 10^{-3} = \frac{1}{2} k (v_{GS} - V_T)^3 = (v_{GS} - V_T)^3$$

$V_{GS} = \pm 1 + V_T = \pm 1 + 1 \text{ in saturation, } V_{GS} > V_T = 1$

$\therefore \ V_{GD} = 2$

$V_{GS} + 1 k(i_D) = V_i = 2 + 1 = 3 \text{ V}$

$V_i = 3 \text{ V}$
(2B) Validate or disprove the assumption that the MOSFET operates in the saturation region for the proposed operating condition.

For saturation: \( V_{GS} > V_T = 1 \) is satisfied

Also \( V_{DS} > V_{GS} - V_T \) to operate in saturation

From Figure: \( V_{DS} = V_G - C_D(100) = 4 \text{ Volts} \)

\( 4 > V_{GS} - V_T = 2 - 1 = 1 \)

MOSFET saturation region satisfied? Justification?

Yes

\[ V_{GS} = 2 > V_T = 1 \]

\[ V_{DS} = 4 > V_{GS} - V_T = 1 \]
(2C) Given the small-signal model for the MOSFET shown in Figure 3, draw the small-signal model for the amplifier. Make sure to label all important circuit parameters and variables.

![Small-signal model for MOSFET](image)

**Figure 3**

Draw the small-signal circuit model for the amplifier

![Small-signal circuit model for amplifier](image)
(2D) Calculate the small-signal voltage gain for the amplifier and express it in terms of $g_m$, $R_D$, and $R_S$.

$$V_o = -g_m R_D V_{gs}$$

$$V_{gs} = \frac{V_L}{1 + g_m R_S}$$

$$\frac{V_o}{V_i} = \frac{-g_m R_D}{1 + g_m R_S}$$
Problem 3  (25 Points)

For the circuits below, please find expressions for the specified voltage over the indicated time ranges in terms of the circuit parameters. Plot the waveform on the provided axes, and clearly identify the key parameters in your graph.

(3A) Consider the circuit of Fig. 4. The switch is open for $t < 0$, closed for $0 \leq t < t_1$, and open for $t \geq t_1$, where $t_1 = 3RC$. Find and plot the voltage $v_C(t)$.

\[ v_C = \begin{cases} 0, & t < 0 \\ \frac{RI}{2} \left( 1 - e^{-\frac{2t}{RC}} \right), & 0 \leq t < t_1 \\ \frac{RI}{2} (1 - e^{-\frac{2t}{RC}}), & t \geq t_1 \end{cases} \]

Resonance "seen" by capacitor with $I$ turned off

\[ R_{eq_{\text{initial}}} = \frac{R}{2}, \quad C = \frac{RC}{2} \]

\[ v_{c_{\text{initial}}} = 0, \quad V_{\text{initial}} = \frac{RI}{2} (1 - e^{-\frac{2t}{RC}}) \]

For $t = t_1$, $t = 2RC$

\[ V_{c_{\text{final}}} = \frac{RI}{2} (1 - e^{-\frac{2t}{RC}}) \]

\[ V_{c_{\text{final}}} = \frac{RI}{2} (1 - e^{-\frac{2t}{RC}}) - \frac{(t - 3RC)}{RC} \]

\[ v_{c_{\text{final}}} = 0 \]

\[ v_{c_{\text{final}}} = \frac{RI}{2} (1 - e^{-\frac{2t}{RC}}) \]
Consider the circuit of Figure 3.3 in which $a > 1$. The switch is open for $0 < t < t_1$. Find and plot the voltage $v_c(t)$.

\[ v_c(t), t < 0 = \begin{cases} 0 \\ \frac{RI}{2} \left(1 - e^{-\frac{RI}{RC}}\right) \\ \frac{RI}{2} \left(1 - e^{-\frac{RI}{RC}}\right)e^{-\frac{(t-t_1)RC}{RC}} \end{cases} \]

Plot $v_c(t)$ over all time, indicating important waveform parameters.

\[ v_c(t), 0 \leq t < t_1 = \frac{RI}{2} \left(1 - e^{-\frac{RI}{RC}}\right) \]

\[ v_c(t), t \geq t_1 = \frac{RI}{2} \left(1 - e^{-\frac{RI}{RC}}\right)e^{-\frac{(t-t_1)RC}{RC}} \]
(3B) Consider the circuit of Figure 5, in which $\alpha > -1$. The switch is open for $t < 0$, and closed for $t \geq 0$. $v_C(0^-) = V_0$. Find and plot the voltage $v_1(t)$.

![Diagram of the circuit](image)

Figure 5

$$v_1(t) = N_{\text{FINAL}} - N_{\text{INITIAL}} - t/\alpha$$

Find $R_{\text{IN}}$ at capacitor terminals

$$R_{\text{IN}} = R_1 + l_i$$

$$N_{\text{IN}} = R_{\text{IN}} l_i = \frac{R_1 l_i}{1 + \alpha}$$

Then $V = \frac{R_C}{1 + \alpha}$

$N_{\text{FINAL}} = N_{\text{INITIAL}} = V_0$ (all finite sources)

$N_{\text{FINAL}} = 0$ (no energy source at $t = 0$)

$v_1(t) = V_0 e^{-t/(1+\alpha)}$ for $t > 0$
The engineers at Belllyup Labs have asked your assistance in developing a small-signal circuit model for the Dualistor. Derive a small-signal circuit model for the Dualistor with small-signal short open about a bias point in a 1. Draw and label the small-signal circuit, making sure to label all important circuit variables and parameters.

\[ v_i(t), \ t < 0 = 0 \]
\[ v_i(t), \ t \geq 0 = v_0 \frac{-t(t+\alpha)}{RC} \]

Plot \(v_i(t)\) over all time, indicating important waveform parameters.
Problem 4 (13 Points)

After years of research, the FUBAR division of Bellyup Labs has created a new three-terminal semiconductor device that they name the "Dualistor". The proposed symbol for the device is shown in Fig. 6, along with a large-signal circuit model that the Bellyup researchers tell you is valid over the range of interest.

![Image of Dualistor symbol and large-signal circuit model]

**Figure 6**

The researchers at Bellyup Labs have asked your assistance in developing a small-signal circuit model for the Dualistor. Derive a small-signal circuit model for the Dualistor operating about a bias point $i_F = I_F$. Draw and label the small-signal circuit, making sure to indicate all important terminal variables and parameters.

Taylor series for $V_{RB} = C(i_F)^2$ about $i_F = I_F + \delta i_F$

$V_{RB} = C(I_F)^2 + 2C I_F \delta i_F + \frac{1}{2} (2C I_F)^2 \delta i_F^2$

Assume $\delta i_F \ll 1$
Draw and label the small-signal circuit model for the Dualistor. Make sure to provide values for any circuit parameters that are introduced.
Figure 7 shows a relay driver circuit. The low-power switch $S$ is used to activate and deactivate the relay (which can control much more power). The relay is considered “activated” whenever the relay current $i_L$ exceeds 25 mA, and is “deactivated” otherwise. The relay is modeled as the series connection of a resistor $R_R = 100 \, \Omega$ and an inductor $L_R = 200 \, \mu\text{H}$. To protect the switch $S$ driving the relay, a resistor $R_F = 100 \, \Omega$ is placed across the relay as shown.

**Figure 7**

(5A) What will be the delay between the time the switch $S$ is closed (after being open for a long time) and the time the relay is “activated”?

\[
L_L = L_L^{\text{initial}} + (L_L^{\text{initial}} - L_L^{\text{final}}) e^{-t/T} \quad (\text{2})
\]

When the resistance seen from inductor terminals with voltage switch off $= R_R$

\[
T = L / R_R = 2\times10^{-6} \quad (\text{2})
\]

\[
L_{L^{\text{initial}}} = 5 \times 10^{-3} \quad (\text{2})
\]

\[
L_{L^{\text{final}}} = 5 \times 10^{-3} \quad (\text{2})
\]

\[
I_L = 5 \times 10^{-3} (1 - e^{-t/T}) \quad (\text{2})
\]

Current required $25 \times 10^{-3} = 5 \times 10^{-3} (1 - e^{-t/T})$

\[
0.5 = 1 - e^{-t/T} \quad (\text{2})
\]

\[
e^{-t/T} = 0.5 \quad (\text{2})
\]

\[
t/T = \ln(0.5) = 0.693 \quad (\text{2})
\]

\[
t/T = 1.38 \times 10^{-6} \quad (\text{2})
\]

Activation delay $= 1.38 \times 10^{-6}$ seconds
(5B) After being open for a long time, the switch $S$ is closed at $t = 0$, then opened at $t = 10 \mu s$. On the axes provided, plot the switch voltage $v_{sw}$ and the switch current $i_{sw}$ from $t = -5 \mu s$ to $15 \mu s$. Clearly label and specify (numerically) all steady-state levels and time constants.

For $t < 0$, $v_{sw} = 5V$, $i_{sw} = 0$
For $0 \leq t < 10\mu s$, from Part A, $L(t) = 50 \times 10^{-3} (1 - \frac{t}{4 \times 10^{-6}})$
Switch is closed, $v_{sw} = 0$ during this period
$L(t) = \frac{d}{dt} + \frac{i_t}{L(t)} = 50 \times 10^{-3} (2 - \frac{t}{4 \times 10^{-6}})$

For $t > 10\mu s$, switch opens, current through inductor is continuous (all finite sources) and flows through $R_f$.
Then $v_{sw} (t=10\mu s) = R_f i_L (10\mu s) + 5 = 9.96$
Inductor = $50\mu s$ $R_f$ and $R_f = \frac{L}{R_f} (R_f + 5) = \frac{L}{R_f} (R_f + 5) = 1 \times 10^{-6}$
$L(t) = 0$, $v_{sw} = 5 + C_L R_f = 5 + 4.96 \frac{t}{(2-10 \times 10^{-6}) / 1 \times 10^{-6}}$

\[ t_1, t_2 \]

$\text{step to 50 mA}$
$\text{final 1 mA}$
$\text{step to 9.96}$
$\text{finish at 5}$