February 23, 2005 - Quiz #1

Name: SOLUTIONS [ VERSION 1 ]

Section (circle appropriate one): 10 AM, 11 AM, noon, 1 PM

Problem 1 (10 points):

Problem 2 (10 points):

Total (20 points):

General guidelines (please read carefully before starting):

• Make sure to write your name on the space designated above.
• Please circle your recitation hour above.
• Closed book.
• All answers should be given in the space provided. Please do not turn in any extra material.
• You have 50 minutes to complete your quiz.
• Partial credit will be given for setting up problems without calculations. NO credit will be given for answers without reasons.
1. (10 points) Consider the following circuit.

![Circuit Diagram]

This problem is about deriving Thevenin and Norton equivalent circuits at the output terminals marked $v_o$.

a) (2 points) Calculate the open circuit voltage at the output terminals.

\[
V_{o+} = \frac{3}{4} (12 \, V) = 9 \, V \\
V_{o-} = \frac{2}{8} (12 \, V) = 3 \, V
\]

\[
\therefore \quad V_o = V_{o+} - V_{o-} = 6 \, V
\]
b) *(2 points)* Calculate the Thevenin resistance looking into the output terminals.

*Turn off independent sources:*

\[
\begin{align*}
R_1, R_2, & \quad R_3, R_4 \\
\text{\( R_{TH} \)} & \quad \text{\( R_{TH} \)}
\end{align*}
\]

\[
R_{TH} = (R_1 \ || \ R_2) + (R_3 \ || \ R_4) = \frac{3}{4} + \frac{12}{8} = 2.25 \Omega
\]
c) (2 points) Calculate the short circuit current at the output terminals.

\[ V_S = 12 \, \text{V} \]

\[ R_1 = 1 \, \Omega \]
\[ R_2 = 3 \, \Omega \]
\[ R_3 = 6 \, \Omega \]
\[ R_4 = 2 \, \Omega \]

\[ I_{SC} = -I_N \]

\[ I_{SC} = -\frac{V_{TH}}{R_{TH}} = -2.67 \, \text{A} \]
d) (2 points) Calculate the values for the elements of the Thevenin and Norton equivalent circuit models, as shown below (units expected).

\[ V_{TH} = 6 \text{ V} \]
\[ R_{TH} = 2.25 \Omega \]
\[ I_N = 2.67 \text{ A} \]
\[ R_N = 2.25 \Omega \]
e) (2 points) Now solve for $I_L$ in the circuit below, if $V_L = 9 \, V$.

![Circuit Diagram]

Replace circuit with Thevenin Equivalent:

![Thevenin Equivalent Diagram]

\[ I_L = \frac{6 - 9}{2.25} = -1.33 \, A \]
2. (10 points) Consider a non-linear device (known as "W") with quadratic I-V characteristics given by:

\[
\begin{align*}
  i_W &= 0.25v_W^2 \quad \text{for } v_W \geq 0 \\
  i_W &= 0 \quad \text{for } v_W \leq 0
\end{align*}
\]

Consider this device being used in the following circuit:

\[I_S=0.5 \text{ A} \quad i_s=1 \mu\text{A} \quad 4 \Omega \quad v_W=V_W+v_W\]

\(I_S\) is a DC current source and \(i_s\) is a small-signal current source.

a) (5 points) Calculate the DC voltage, \(V_W\), across the W device when \(i_s = 0\).

We know from orientation of \(I_S\) that \(V_W > 0\)

\[
\Rightarrow 0.5 = \frac{V_W}{4} + 0.25V_W^2
\]

\[
\Rightarrow 0 = 0.25V_W^2 + \frac{V_W}{4} - 0.5 = 0
\]

\[
\Rightarrow V_W = 1 \Rightarrow -2 \text{ V}
\]

But we know \(V_W > 0\), so \(V_W = 1 \text{ V}\)
b) (5 points) With the W device biased with $I_S$, calculate the small-signal voltage, $v_w$, across the W device in response to the small-signal current source $i_w$.

$$\text{Since } v_w > 0, \quad i'_w = 0.25 v_w^2$$

$$\Rightarrow \quad \frac{2 i'_w}{v_w} \bigg|_{v_w = v_w'} = v_w = i'_w$$

$$\Rightarrow \quad r_w = \frac{v_w}{i'_w} = \frac{1}{0.5} = 2 \Omega$$

$$\therefore \quad v_w = i'_w \cdot (4/2) = 1.33 \mu V$$