Enter all your work and your answers directly in the spaces provided on the printed pages. Answers must be derived or explained, not simply written down. The quiz is closed book, but calculators are allowed.

The quiz contains 8 pages including the cover sheet. Make sure that your quiz contains all 8 pages and that you hand in all 8 pages.

Parts of the two problems can be worked independently, so be sure not to spend too much time on any one part!
Problem 1. (6 points)

![Diagram of a crude digital thermometer](image)

The figure shows a schematic diagram of a crude digital thermometer that operates for temperatures between 0° and 20° C. An analogue signal proportional to temperature in this range is digitized by a 2-bit A to D converter. The output of the converter is connected to a logic network with 4 outputs. Each output connects to a light emitting diode (through an appropriate amplifier).

When the temperature is between 0 and 5° C the digitizer output is D1D2=00 and only LED1 is lit, when it is between 5° and 10° C, the digitizer output is D1D2=01 and only LED2 and LED1 are lit, when it is between 10° and 15° C the digitizer output is D1D2=10 and only LED3, LED2 and LED1 are lit, and finally when the temperature is between 15° and 20° C the digitizer output is D1D2=11 and all LED's are lit. The task in this problem is to design the logic network and realize it using MOSFET gates.

a) Find a logical expression for the LED2 input in terms of the digitizer output D1D2

\[
\text{LED}_2 = \overline{D}_1 \bar{D}_2 + D_1 \bar{D}_2 + D_1 D_2 = D_1 + D_2
\]
Problem 1. (Continued)

b) Draw a MOSFET realization of your result for LED₂ using only the above Inverter and NOR circuits. Assume that two inputs proportional to D₁ and D₂ respectively are available.

Looking at (c), we see \( \text{LED}_2 = D_1 + D_2 = \overline{D_1 + D_2} \)

[Diagrams of circuits]
Problem 1. (Continued)

c) The MOSFET logic must satisfy the static discipline: $V_{OL} = 1 \text{ V}$, $V_{IL} = 2 \text{ V}$, $V_{IH} = 4 \text{ V}$ and $V_{OH} = 5 \text{ V}$. What ranges of $R_{PU}$ and $V_T$ are allowed, assuming $R_{ON} = 20 \text{ Ohms}$?

To recognize input, we must put $V_T$ between $V_{IL}$ and $V_{IH}$. Also, low output must be less than $V_{OL}$.

$$\Rightarrow \frac{R_{ON}}{R_{ON} + R_{PU}} \quad (5) < 1$$

$$\Rightarrow 5 \cdot R_{ON} < R_{ON} + R_{PU}$$

$$\Rightarrow R_{PU} > 4 \cdot R_{ON}$$

$$R_{PU} > 80 \text{ Ohms}$$

$$2 < V_T < 4 \text{ Volts}$$
Problem 2. (14 points)

In the saturation region, the MOSFET shown in the above circuit has an \( i_D - v_{DS} \) relation given by

\[
i_D = \frac{K}{2} (v_{GS} - V_T)^3
\]

where \( K = \frac{1}{2} \text{ mA/V}^3 \) and \( V_T = 2 \text{ V} \). Note the cubic dependence in the \( i_D - v_{DS} \) relation.

a) Find the relation between \( R_1 \) and \( R_2 \) that will fix the output bias voltage at \( V_{OUT} = 6 \text{ V} \). (For this bias the MOSFET will be in the saturation region.)

\[
\begin{align*}
\Delta v_{out} &= 10 - i_D R_L = 10 - \frac{K R_L}{2} (v_{GS} - V_T)^2 \\
\Rightarrow \quad v_{GS} &= \left[ \left( 10 - \Delta v_{out} \right) \left( \frac{2}{K R_L} \right) \right]^{1/2} + V_T = \frac{R_1}{R_1 + R_2} (10)
\end{align*}
\]

\[
\Rightarrow \quad \frac{R_1}{R_1 + R_2} = \frac{2}{5}
\]

\[
\Rightarrow \quad 5 R_1 = 2 R_1 + 2 R_2
\]

\[
\Rightarrow \quad \frac{R_1}{R_2} = \frac{2}{3}
\]

\[
R_1/R_2 = \frac{2}{3}
\]
b) A small signal source is added to the circuit as shown in the above figure. Draw a small-signal model for the circuit that can be used to analyze its small signal gain $G = \frac{v_{out}}{v_{in}}$. Be sure that all parameter values are labeled and proper units are given. Your model should include a dependent source relating $i_d$ to $v_{gs}$.

$$g_m = \frac{v_{gs}}{v_{ds}}$$

and has units $\frac{A}{V}$. 

$$v_{ds} = v_{gs}$$
c) Calculate the small signal gain \( G = \frac{v_{out}}{v_{in}} \) for this circuit. (You may either use your small-signal circuit determined in b) or linearize the large signal \( v_{OUT} \) vs. \( v_{IN} \) relation to answer this part. In any case, a numerical result is asked for.) 

\[
\begin{align*}
g_m &= \frac{\partial v_{out}}{\partial v_{in}} \\
v_{inv} &= \frac{3k}{2} (v_{GS} - V_T)^2 = 3 \frac{mA}{V} \\
\Rightarrow v_{inv} &= -g_m v_{GS} R_L = g_m v_{in} R_L \\
\Rightarrow G &= g_m R_L = 6
\end{align*}
\]
Problem 2. (Continued)

d) The small signal source is given by \( v_{\text{in}} = Vu(t) \) where \( u(t) \) is the unit step function. Let the gate-to-source capacitance of the MOSFET be \( C_{\text{GS}} \). Find the small signal response \( v_{\text{out}}(t) \). Express your answer in terms of \( C_{\text{GS}}, R_1, R_2, G \) and \( V \).

\[
\text{Small signal model looks like}
\]

\[
\Rightarrow v_{\text{out}} = -g_m R_2 V_{\text{gs}} = -G V_{\text{gs}}
\]

But \( V_{\text{gs}}(t) = V g(t) - V_s(t) = V g(t) - V \) for \( t > 0 \) and \( V g(t) = V e^{-t/\tau} \) where \( \tau = C_{\text{gs}} (R_1 / R_2) \)

\[
\Rightarrow V_{\text{gs}}(t) = V (e^{-t/\tau} - 1)
\]

\[
\Rightarrow v_{\text{out}}(t) = GV (1 - e^{-t/\tau})
\]

\[
v_{\text{out}} = GV (1 - e^{-t/\tau}), \quad \tau = C_{\text{gs}} (R_1 / R_2)
\]