Exercise 7-1: Using one 1-mH inductor and two resistors, construct a two-port network that has the following zero-state response to a 1-V step input as shown below. Provide a diagram of the network, and specify the values of the two resistors.

Exercise 7-2: All networks shown below begin operation at $t = 0^-$ with zero capacitor voltage or zero inductor current. That is, all states are zero at $t = 0^-$. For each network, find the network state, that is the capacitor voltage or inductor current, at both $t = 0^+$ and $t = \infty$. Also find the time constant by which the network state goes from its initial value at $t = 0^+$ to its final value at $t = \infty$. Finally, without actually solving an appropriate differential equation, find the network state for each network for $0^+ \leq t \leq \infty$.

Reminder: the mid-term exam for 6.002 is on Thursday October 29 from 7:00 PM until 9:00 PM in Walker Memorial. The exam will cover the material in Homeworks #1 through #6. Also, there will be no recitations on Friday October 30 due to the evening mid-term exam.
Problem 7-1: This problem explores the basis for treating an initial condition as a network input. Doing so allows the network response to the initial condition to be superimposed on the response to other inputs. Thus, this problem explores the basis behind creating a network response through the superposition of a ZIR and a ZSR.

(a) Network #1 is simply a capacitor with the initial condition of $v_0$ at $t = 0$. For this network, find $v(t)$ as a function of $i(t)$ for $t \geq 0$.

(b) The capacitor in Network #2 has zero initial condition at $t = 0$. For this network, find $v(t)$ as a function of $i(t)$ for $t \geq 0$.

(c) What must be the relation between $v_0$ in Network #1 and $V_0$ in Network #2 for the terminal relations between $v(t)$ and $i(t)$ to be identical, thereby permitting one network to substitute for the other. With this substitution, the initial capacitor voltage may be treated as a source, thereby allowing superposition.

(d) Network #3 is simply an inductor with the initial condition of $i_0$ at $t = 0$. For this network, find $i(t)$ as a function of $v(t)$ for $t \geq 0$.

(e) Construct a network containing an inductor and a current source which has the same relation between $i(t)$ and $v(t)$ for $t \geq 0$ as does Network #3. The inductor in the new network must have a zero initial condition.

Problem 7-2: This problem is a continuation of Problem 6-1, and explores a slightly different method of solution. You should think about which method you find easiest.

(a) Find the zero-input response for Problem 6-1. That is, find the response to $i_0$ with $V_0 = 0$.

(b) Find the zero-state response for Problem 6-1. That is, find the response to $V_0$ with $i_0 = 0$.

(c) Using superposition, add the two responses to obtain the total response. This should yield the same response you found in Problem 6-1.
Problem 7-3:  This problem examines the power dissipated by a small digital logic circuit. The circuit comprises a series-connected inverter and NOR gate as shown below. The circuit has two inputs, A and B, and one output, Z. The inputs are assumed to be periodic as shown below. Assume that $R_{ON}$ for each MOSFET is zero.

(a) Sketch and clearly label the waveform for the output Z for $0 \leq t \leq T_1$. In doing so, assume that $C_G$ and $C_L$ are both zero.

(b) Derive the time-average static power consumed by the circuit in terms of $V_S$, $R_L$, $T_1$, $T_2$, $T_3$ and $T_4$. Here, time-average power is defined as the total energy dissipated by the gate during the period $0 \leq t \leq T_4$ divided by $T_4$.

(c) Now assume that $C_G$ and $C_L$ are nonzero. Derive the time-average dynamic power consumed by the circuit in terms of $V_S$, $R_L$, $C_G$, $C_L$, $T_1$, $T_2$, $T_3$ and $T_4$. In doing so, assume that the circuit time constants are all much smaller than $T_1$, $T_2 - T_1$, $T_3 - T_2$ and $T_4 - T_3$.

(d) Evaluate the time-average static and dynamic powers for $V_S = 5$ V, $R_L = 10$ kΩ, $C_G = 100$ fF, $C_L = 1$ pF, $T_1 = 100$ ns, $T_2 = 200$ ns, $T_3 = 300$ ns and $T_4 = 600$ ns.

(e) By what percentage does the total time-average power consumption drop if the power supply voltage $V_S$ drops by 30%?
Problem 7-4: This problem studies the modeling of lumped-parameter thermal systems with electrical networks comprised of sources, resistors and capacitors. Here, voltage represents temperature, and current represents the flow of heat, or the flow of thermal energy.

(a) As a specific example, consider the thermal behavior of a house modeled as a single thermal unit. Heat is supplied to the house by its furnace, which is modeled by a current source. The heat is stored in the heat capacity of the house, which is modeled by a capacitor. Heat conducts from the house to the air outside through the windows, outer walls and roof of the house; heat conduction through the foundation is ignored here. The path of heat conduction through the walls and roof is modeled by a resistor, and the path of heat conduction through the windows is modeled by another resistor. The outside air temperature is set by a voltage source from the reference of 0 °C. Following this description, derive a three-node electrical network which models the thermal behavior of the house. Two node voltages should represent the temperatures inside and outside the house, respectively. The third node should be the ground reference node at 0 °C. Clearly identify each node and the thermal correspondence of each electrical element. In the electrical model, let a 1-V difference between nodes correspond to a 1-°C temperature rise, and let a 1-A branch current correspond to a 1-W heat flow.

(b) Let the house be 8 m wide, 20 m long and 8 m high, and approximate its roof as being flat. Further, let the house have 36 windows each with an area of 1 m². Given that the thermal conductivity of the walls and roof is 0.2 W/m²/°C, and the thermal conductivity of the windows is 2.0 W/m²/°C, determine the electrical resistance in Ohms of the two resistors in the model.

(c) If the outside air temperature is 0 °C, and the furnace is turned off, then the temperature inside the house is observed fall from 20 °C to 10 °C in 10 hours. Determine the electrical capacitance in Farads of the capacitor in the model.

(d) Given an outside air temperature of -10 °C, and a temperature inside the house of 20 °C, determine the rate in Watts at which the furnace must supply heat to the house in order to maintain the temperature difference in steady state.

(e) Overnight, the temperature inside the house is allowed to cool to 10 °C. In the morning, the outside air temperature is 0 °C. Determine the rate in Watts at which the furnace must supply heat in order to raise the temperature inside the house to 20 °C in 5 minutes.

(f) Assume that the house is in the thermal steady state described in Part (d). At $t = 0$, the outside air temperature steps to 0 °C, while the furnace continues to supply heat at the same rate. Determine the temperature in the house for $t \geq 0$. 

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