(D) Following Part A, let \( v_I(t) = V_I e^{j\omega t} \). Also, let \( v_O(t) = \hat{V}_O e^{j\omega t} \) where \( \hat{V}_O \) is a complex function of \( \omega \). With these substitutions, use the differential equation to find \( \hat{V}_O \).

(E) Following Parts A and B, find \( V_O \) and \( \phi \) from \( \hat{V}_O \), both as functions of \( \omega \).

(F) Sketch and clearly label the dependence of \( \log(V_O/V_I) \) and \( \phi \) on \( \log(\omega \tau) \), where \( \tau \) is the time constant of the circuit given below. Identify the low- and high-frequency asymptotes on the sketch.
Problem 8.3: In the network shown below, the inductor and capacitor have zero states prior to $t = 0$. At $t = 0$, a step in voltage from 0 to $V_O$ is applied by the voltage source as shown.

(A) Find $v_C$, $v_L$, $v_R$, $i$ and $\frac{di}{dt}$ at $t = 0$.
(B) Argue that $i = 0$ at $t = \infty$ so that $i(t)$ has no constant component.
(C) Find a second-order differential equation which describes the behavior of $i(t)$ for $t \geq 0$.
(D) Following (B) the current $i(t)$ takes the form $i(t) = I \sin(\omega t + \phi)e^{-\alpha t}$. Find $I$, $\omega$, $\phi$ and $\alpha$. Hint: first find $\omega$ and $\alpha$ from the differential equation, and then find $I$ and $\phi$ from the initial conditions; alternatively, solve this problem by any method you wish.
(E) Suppose that the input is a voltage impulse with area $\Lambda_O$ where $\Lambda_O = \tau V_O$. $V_O$ is the amplitude of the voltage step shown below, and $\tau$ is a given time constant. Find the response of the network shown below to the impulse. Hint: before solving this problem directly, consider the relation between step and impulse responses.

Save a copy of your answers to this problem. They will be useful during the pre-lab exercises for Lab #3.

Problem 8.4: The network shown below is driven in steady state by the sinusoidal input voltage $v_I(t) = V_I \cos(\omega t)$. The output of the network is the voltage $v_O(t)$, which takes the form $v_O(t) = V_O \cos(\omega t + \phi)$. Find $V_O$ and $\phi$ as functions of $\omega$ as follows.

(A) Using the Taylor Series expansions for $e^x$, $\cos(x)$ and $\sin(x)$, show that $e^{ix} = \cos(x) + j\sin(x)$. Following this, recognize that $\cos(x) = \Re\{e^{ix}\}$.
(B) Show that $A + Bj = \sqrt{A^2 + B^2} e^{j\arctan(B/A)}$. Thus, the magnitude and phase of $A + Bj$ are $\sqrt{A^2 + B^2}$ and $\arctan(B/A)$, respectively.
(C) Find a differential equation that can be solved for $v_O(t)$. 
**Problem 8.1:** The network shown below includes two switches: #1 and #2. Prior to \( t = 0 \), both switches are closed, and the capacitor voltage \( v(t) \) and inductor current \( i(t) \) are both zero.

(A) At \( t = 0 \), Switch #1 opens, and it remains open until \( t = T_1 \). Find \( v(t) \) and \( i(t) \) for \( 0 \leq t \leq T_1 \).

(B) At \( t = T_1 \), Switch #1 closes as Switch #2 simultaneously opens. They remain in these states until \( v(t) \) goes to zero, at which time Switch #2 closes. Define the time at which \( v(t) \) goes to zero as \( t = T_2 \). Determine \( T_2 \), and find \( v(t) \) and \( i(t) \) for \( T_1 \leq t \leq T_2 \).

(C) Both switches remain closed until \( t = T_3 \). Find \( v(t) \) and \( i(t) \) for \( T_2 \leq t \leq T_3 \).

(D) At \( t = T_3 \), Switch #1 again opens, and it remains open until \( t = T_4 \). Find \( v(t) \) and \( i(t) \) for \( T_3 \leq t \leq T_4 \).

(E) Finally, at \( t = T_4 \), Switch #1 closes as Switch #2 again simultaneously opens. They remain in these states until \( v(t) \) again goes to zero, at which time Switch #2 closes. Define the time at which \( v(t) \) again goes to zero as \( T_5 \). Determine \( T_5 \), and find \( v(t) \) and \( i(t) \) for \( T_4 \leq t \leq T_5 \).

(F) Sketch and clearly label \( v(t) \) and \( i(t) \) for \( 0 \leq t \leq T_5 \).

\[ \text{\includegraphics[width=0.5\textwidth]{network_diagram.png}} \]

**Problem 8.2:** This problem is a continuation of Problem 8.1. It explores the use of energy conservation to analyze the operation of the network described therein.

(A) Determine the energy stored in the capacitor at \( t = T_1 \).

(B) The energy stored in the capacitor at \( t = T_1 \) is transferred to the inductor at \( t = T_2 \). Use this fact to determine \( i(T_2) \). This answer should match the answer to Part B of Problem 8.1.

(C) Determine the energy stored in the capacitor at \( t = T_4 \).

(D) Use energy conservation to determine the energy stored in the inductor at \( t = T_5 \), and then determine \( i(T_5) \). This answer should match the answer to Part E of Problem 8.1.
Exercise 8.1:  All networks shown below begin operation at $t = 0^-$ with zero capacitor voltage or zero inductor current. That is, all states are zero at $t = 0^-$. For each network, find the network state, that is the capacitor voltage or inductor current, at both $t = 0^+$ and $t = \infty$. Also find the time constant by which the network state goes from its initial value at $t = 0^+$ to its final value at $t = \infty$. Finally, without actually solving an appropriate differential equation, find the network state for each network for $0^+ \leq t \leq \infty$.

Exercise 8.2:  Using one 1-µF capacitor and three resistors, construct a two-port network that has the following zero-state response to a 3-V step input as shown below. Provide a diagram of the network, and specify the values of the three resistors.