Problem 8.1: The inductor in the LC circuit below has an initial current \( i = I \) Amperes. At \( t = 0 \) the switch opens.

\[
\begin{array}{c}
\text{+} \\
v \\
C \\
\text{sw} \\
L \\
\text{-} \\
\end{array}
\quad \begin{array}{c}
i \\
C = 0.1 \mu F \\
L = 40 \text{ mH} \\
\end{array}
\]

(A) Determine the natural frequency and the period of the oscillation which occurs for \( t > 0 \). Specify units!

(B) Write a differential equation for the current \( i \) or the voltage \( v \) which applies for \( t > 0 \).

(C) Solve this equation, apply the indicated initial conditions, and write expressions for \( i(t) \) and \( v(t) \) for \( t > 0 \).

Problem 8.2: The circuit of Problem 8.1 is modified by adding a high-value parallel resistor of conductance \( G \). The initial inductor current is \( i = I \) and the switch opens at \( t = 0 \).

\[
\begin{array}{c}
\text{+} \\
v \\
G \\
C \\
\text{sw} \\
L \\
\text{-} \\
\end{array}
\quad \begin{array}{c}
i \\
C = 0.1 \mu F \\
L = 40 \text{ mH} \\
G = 5 \times 10^{-4} \text{ Mhos} \\
\end{array}
\]

(A) Write a differential equation for \( v \) which applies for \( t > 0 \).

(B) Determine the characteristic equation for the circuit, the roots of which are the natural frequencies.

(C) Determine the damping factor \( \alpha \), the natural frequency \( \omega \), and the \( Q \) of the lightly damped oscillator.
**Problem 8.3:** The circuits shown below are driven by sinusoids. In each case express the indicated variables as functions of time, i.e. \( f(t) = A \cos(\omega t + \phi) \) and write expressions for the magnitudes and the phases.

**Hint:** Use a complex exponential as the driving function.

(A)

![Circuit Diagram A](image)

\[ i(t) = ? \]

\[ v_L = ? \]

(B)

![Circuit Diagram B](image)

\[ i(t) = ? \]

\[ v_C = ? \]

**Problem 8.4:** The circuits show below, which are driven by sinusoids, have the indicated responses.

I)

\[ i_S(t) = I \cos(\omega t) \]

\[ v_L(t) = I \frac{\omega L}{\sqrt{1+(\frac{\omega}{R})^2}} \cos(\omega t + \frac{\pi}{2} - \tan^{-1}(\frac{\omega L}{R})) \]

II)

\[ v_S(t) = V \cos(\omega t) \]

\[ i(t) = V \frac{\omega C}{\sqrt{1+(\omega RC)^2}} \cos(\omega t + \frac{\pi}{2} - \tan^{-1}(\omega RC)) \]

(A) Focus first on circuit I). Recall that the magnitude of the impedance of an inductor varies as \( \omega L \), and note that the circuit has the form of a current divider.

At the limit of very low frequencies \( (\omega << \frac{R}{L}) \) reason from the circuit alone to determine an approximate value for the magnitude of \( v_L \). Verify your answer by using the given response.

(B) At the limit of very high frequencies \( (\omega >> \frac{R}{L}) \) use circuit reasoning to determine an approximate value for the magnitude of \( v_L \).

(C) At what frequency does the current \( i_S(t) \) divide equally in magnitude between \( R \) and \( L \)?

(D) At this frequency, what are the magnitude and phase of the response?
The following questions are similar to A) - D) above but apply to circuit II). Note that it has the form of a voltage divider and recall that the magnitude of the impedance of a capacitor varies as $\frac{1}{\omega C}$.

(E) For the repeat of Part A), what is the condition for the low frequency limit that corresponds to $(\omega << \frac{1}{R})$? Reason from the circuit alone to determine approximate values for the magnitudes of $v_C$ and $i$ at very low frequencies. Verify your answers by using the given responses.

(F) For the repeat of Part B), what is the condition for the high frequency limit that corresponds to $(\omega >> \frac{1}{R})$? Reason from the circuit along to determine approximate values for the magnitudes of $v_C$ and $i$ at very high frequencies. Verify your results by using the given responses.

(G) At what frequency does the voltage $v_S(t)$ divide equally in magnitude between $R$ and $C$?

(H) At this frequency what are the magnitudes and phases of the responses?