Notes for 6.002 Lecture #17, April 10, 2003

Lab #3 Next week
Lab #4 Three weeks later (3/04 May 5) Term ends on May 15

More Sinusoidal Steady State:

Argument once again:
1) Actual Excitation is Sinusoidal e.g. \( v \cos(\omega t) \)

\[ V \cos(\omega t) \rightarrow V \exp(j \omega t) \rightarrow V \]

2) Excite instead with \( V \exp(j \omega t) \), the axial part of which is \( V \cos(\omega t) \).

3) Drop \( \exp(j \omega t) \), which appears everywhere in all currents and voltages. Focus on complex amplitudes

4) Use Complex Impedances or Admittances to Represent \( R \) & \( G \).

5) Solve for the desired complex Amplitude using the Algebra of Complex Numbers.

6) Multiply the desired complex amplitude (in polar form) by \( \exp(j \omega t) \) and take the real part, which is the desired actual time function.

Why is Frequency Response Important?

1) Any periodic function of time can be represented as a sum of sinusoids, harmonically related: Fourier Series.

2) Aperiodic functions of time can be represented as an integral of a frequency spectrum: Fourier Integral.

Consider another filter example:

For appropriate element values there will be a region of \( \omega \) in which:

\[ A = \frac{v_2}{V_1} \]

is approx. constant.
AGAIN A VOLTAGE DIVIDER: \( A = \frac{V_2}{V_1} = \frac{R_2 / C_5}{R_2 / C_5 + (R_2 + 1 / C_5)(R_1 + 1 / C_5)} = \frac{R_2}{R_2 + (R_2 C_5 + 1)(R_2 C_5 + 1)} \)

\( A = \frac{R_2/ C_5}{R_2 C_5 + (R_2 C_5 + 1)(R_2 C_5 + 1)} = \frac{R_2 C_5}{R_2 C_5 + (R_2 C_5 + 1)(R_2 C_5 + 1)} \)

FINALLY: \( \bar{A}(s) = \frac{s / R C_2}{s^2 + \left( \frac{1}{R C_2} + \frac{1}{R C_4} + \frac{1}{R C_2} \right) s + \frac{1}{R C_2 R C_4}} = \frac{s / R C_2}{s^2 + \left( \frac{1}{R C_2} + \frac{1}{R C_4} + \frac{1}{R C_2} \right) s + \frac{1}{R C_2 R C_4}} \)

THE DENOMINATOR HAS TWO NEGATIVE REAL ROOFS. THUS \( A(s) \) CAN BE WRITTEN AS:

\( \bar{A}(s) = \frac{s / R C_2}{(s + s_1)(s + s_2)} \)

WHERE \( s_1 << s_2 \) ROOFS ARE \( s_1 = -s_1, s_2 = -s_2 \)

CONSIDER THE ASYMPOTES:

LOW FREQUENCIES: \( s = j \omega \ll s_1 \) AND \( s_2 \)

\( \bar{A}(j \omega) \approx \frac{1}{s / R C_2} \)

THE LINEAR DEPENDANCE ON \( \omega \) TRANSPLANTS ON LOG-LOG COORDINATES TO A STRAIGHT LINE OF SLOPE +1 AND INTERCEPT AT \( \omega = s_1 \)

OF \( 1 / R C_2 \). THE PHASE IS \( \pi / 2 \) OR \( + \pi / 2 \)

HIGH FREQUENCIES: \( s = j \omega >> s_1, s_2 \)

\( \bar{A}(j \omega) \approx \frac{1}{s / R C_2} \)

THE RECIPROCAL DEPENDANCE ON \( \omega \) TRANSPLANTS ON LOG-LOG COORDINATES TO A STRAIGHT LINE OF SLOPE -1 AND INTERCEPT AT \( \omega = s_2 \)

OF \( 1 / R C_2 \). THE PHASE IS \( -\pi / 2 \) OR \( -\pi / 2 \)

MIDDLE FREQUENCIES: \( s_1 < \omega << s_2 \)

\( \bar{A}(j \omega) = \frac{1}{j \omega R C_2} = \frac{1}{j \omega R C_2} \) INDEPENDENT OF FREQUENCY.
**Boole Diagrams**

LoG Scale \[ |\tilde{A}| \]

\[ \frac{1}{B_2 C_2 S_2} \]

**Log Scales**

\[ A \]

For the demo: \( R_1 = 10K \), \( C_1 = 0.1 \mu F \), \( R_2 = 2K \), \( C_2 = 0.01 \mu F \)

Solution of the quadratic yields:

\[ s_1 = 8.2 \times 10^2 \text{ (130 Hz)} \]

\[ s_2 = 6.1 \times 10^4 \text{ (9.7 kHz)} \]

**Reprise on Impulses — Difficulties with Dimensions**

Consider:

\[ Q_I(t) \]

\[ u_0(t) \]

\[ U_0(t) = u_0(t) + C \frac{du}{dt} \]

\[ U_0(t) \text{ has dimension of } \frac{1}{\text{sec}} \]

Why?

Impulse is defined as:

\[ u_0(t) = \begin{cases} 0 & t < 0 \\ \infty & t > 0 \end{cases} \]

\[ \int_{-\infty}^{+\infty} u_0(t) dt = 1 \]

This is dimensionless.