Please note that HW#10 is a two week assignment focusing on operational amplifiers. It will receive double weight in the assignment of homework grades.

**Problem 10.1:** Most circuits which employ operational amplifiers rely on negative (error-correcting) feedback. This problem explores several of the consequences of such feedback in an abstract form. Understanding a simple abstract example illustrating some of the consequences of feedback may make the operational amplifier easier to understand.

The element A is an amplifying element which has very high gain. However, the gain varies greatly from unit to unit and is strongly temperature dependent.

The element B is a variable attenuator. Its output $y' = \beta y$ where $\beta < 1$.

The element $\sum$ is a summing element. Its output $\delta$ is the difference between the input $x$ and the fed-back variable $y'$, that is, $\delta = x - y'$.

The summing element compares the input with a fraction $\beta$ of the output. Thus, in the ideal situation, the output would be an amplified replica of the input and the "error variable" would be vanishingly small.

(A) Assume A is a constant - a large positive number. Show that:

$$T = \frac{y}{x} = \frac{A}{1 + A\beta}$$

This is the transfer ratio of the system

Note that if $A/\beta >> 1$

$$T = \frac{A}{1 + A\beta} \approx \frac{1}{\beta}$$

The effect of negative feedback is to yield a closed loop amplification which is determined by the variable attenuator alone and is only weakly dependent on the gain of the unreliable amplifier A.
(B) Assume that $\beta$ is precisely known, but that $A$ may vary over a range of 10:1 because of manufacturing variations, temperature, etc.

$$10^5 < A < 10^6 \quad \beta = 10^{-2}$$

Evaluate $T$ at the extremes of $A$ and determine $\Delta T$, the overall variation in $T$. What is the corresponding fractional change $\frac{\Delta T}{T}$?

A better way of determining the sensitivity of $T$ to changes in $A$ is to:

Evaluate $\frac{dT}{dA}$ and form $\frac{dT}{T} = f(A) \frac{dA}{A}$

Show that $f(A) = \frac{1}{1+A\bar{\beta}}$

What is $\frac{dT}{T}$ for the midrange of the numbers above?

The effect of negative feedback is to reduce the consequences of variations in $A$ by a factor of $\frac{1}{1+A\bar{\beta}} \approx \frac{1}{A\bar{\beta}}$. $A\beta$ is the gain around the closed loop or the closed loop amplification.

(C) Assume that $A$ is a noisy amplifier. That is

$$y = A\delta + N(t)$$

where $A$ is the nominal gain and $N(t)$ is an undesired noise signal.

Determine the noise signal seen at the system output and show that the noise at the output is approximately $\frac{N(t)}{A\bar{\beta}}$ where $A\beta$ is the loop gain.

**Hint:** Think superposition.

For the numbers in Part B), what is the peak-to-peak value of the noise at the output if $N(t) = 5V$ peak-to-peak?

(D) Assume that the amplifying element has frequency limitations. That is to say, represent it as a low-pass filter:

$$\bar{A}(s) = \frac{A_0 s_1}{s + s_1}$$

where $A_0$ and $s_1$ (the bandwidth) are constants.

For low frequencies ($s = j\omega << s_1$) $\bar{A}(j\omega) \approx A_0$, but for frequencies above $s_1$, $\bar{A}(j\omega)$ decreases as $\frac{1}{\omega}$. $s_1$ is the bandwidth of $\bar{A}(j\omega)$.

Show that the bandwidth of the closed loop transfer function $T(s)$ is approximately $A_0\beta s_1$. 
In the operational amplifier, the variables are voltages, the summing function and the gain function are intrinsic and the variable attenuator is provided in the external circuit.

\[ y = A(v_+ - v_-) = A(x - y') \]
\[ y' = \frac{R_1}{R_1 + R_2} \]

**Problem 10.2:** Determine the output voltages $v_O$ of the following circuits. Assume that the operational amplifiers are ideal. That is, the gain $A \rightarrow \infty$. All voltages are defined with respect to ground.

**(A)**

![Circuit A](image)

**(B)**

![Circuit B](image)

**(C)**

![Circuit C](image)

**Problem 10.3:**

The FET is described by the square-law model:

\[ i_D = \begin{cases} \frac{K}{2}(v_{GS} - V_T)^2 & v_{GS} > V_T \\ 0 & v_{GS} < V_T \end{cases} \]

\[ K = 1 \text{mA} V^2 \]

\[ V_T = 1V \]

How does $v_O$ depend on $v_I$? Derive $v_O = v_O(v_I)$. 
**Problem 10.4:** In each of the circuits below, the input is a sinusoid at frequency $\omega \sec^{-1}$. The voltage variables are complex amplitudes. Assume that the op-amps are ideal and that they operate in the linear region. For each circuit, derive the voltage transfer ratio

$$A(s) = \frac{V_O}{V_I}$$

(A) For what range of $\omega$ can this circuit be regarded as an integrator?

(B) Sketch the Bode plot for $\frac{V_O}{V_I}$. Assume $R$ is large enough so the damping factor $\alpha$ is small.

**Problem 10.5:** In the circuit below, the op-amps are ideal and the FET operates in the saturation region and fits the square law model:

$$i_D = \frac{K}{2}(v_{GS} - V_T)^2 \quad \left\{ \begin{array}{l} K = 0.1 \text{ mA/V}^2 \\ V_T = 0 \text{ V} \end{array} \right.$$ 

Find an expression for $v_2$ in terms of $v_1$. 
Problem 10.6: The semiconductor diode in the op-amp circuits below has the usual $i$-$v$ characteristic:

![Diode Circuit](image)

$$i = I_S(e^{qv/kT} - 1) \quad \text{where } I_S \text{ is a constant}$$

(A) $v_I > 0$ and $v_I \gg kT/q$ so that $i \approx I_S e^{qv/kT}$

What is the sign of $v_O$?

Derive an expression for $v_O$ as a function of $v_I$.

(B) $v_I < 0$ and $|v_I| \gg kT/q$ so that $i \approx I_S e^{qv/kT}$

What is the sign of $v_O$?

Derive an expression for $v_O$ as a function of $v_I$.

(C) Let $x$ be a voltage variable which is positive and $y$ be a voltage variable which is negative.

Design a circuit using no more than six op-amps which will produce at its output a voltage proportional to the product $(x)(|y|)$. 