PROBLEM 10.1
A) First, we convert to a system of 3 equations:
\[ y = A \delta \quad \delta = x - y' \quad y' = \beta y \]

Next, simplify to a single equation:
\[ y = A[x - y'] = A[x - \beta y] \]
\[ y = Ax - A\beta y \]
Solving for \( y \) we get:
\[ y = \frac{Ax}{1 + A\beta} \Rightarrow \frac{y}{x} = T = \frac{A}{1 + A\beta} \]

B) \[ T = \frac{A}{1 + A\beta} \Rightarrow \frac{(\text{in table})}{\text{form}} \]

from the table,
\[ \Delta T = 0.09 \]
assuming \( A = 503,000 \) (the midrange value)
\[ \frac{\Delta T}{T} = \frac{0.09}{99.98} \approx 9 \times 10^{-4} \]

\[ dT = \frac{dA(1 + AB) - A(B dA)}{(1 + AB)^2} \Rightarrow \frac{dT}{dA} = \frac{1}{(1 + AB)^2} \]
\[ \frac{dT}{T} = \left[ \frac{1}{(1 + AB)^2} \right] \frac{1 + AB}{A} \frac{dA}{A} = \frac{1}{1 + AB} \frac{dA}{A} = \frac{f(A)}{A} \frac{dA}{A} \]
\[ f(A) = \frac{1}{1 + AB} \]
\[ \frac{dT}{T} = f(A) \frac{dA}{A} = \left( \frac{1}{1 + AB} \right) \frac{dA}{A} \bigg|_{A = 5 \times 10^5} \approx (1.9996 \times 10^{-4}) \frac{dA}{A} \]
c) First, we draw a system diagram:

\[
\begin{array}{c}
\text{x} \\
\delta \\
+ \\
\downarrow \\
A \\
\downarrow \\
\downarrow \\
\beta \\
\uparrow \\
Y \\
\end{array}
\]

Then we write a system of equations and solve for \( y \) as a function of \( N(t) \).

Because we are only interested in \( y \) as a function of \( N(t) \), we can ignore the \( x \) input (superposition): \( x = 0 \)

\[
\begin{align*}
\delta &= 0 - y' = -y' \\
y' &= \beta y \\
y &= A \delta + N(t) \\
y &= A(-y') + N(t) = A(-\beta y) + N(t) \\
y[1 + A\beta] &= N(t) \\
\frac{y}{N(t)} &= \frac{1}{1 + A\beta} \\
\frac{N(t)}{1 + A\beta} &= N(t)
\end{align*}
\]

From Part B: \( A = 5 \times 10^5 \), \( \beta = 0.01 \)

\[
\begin{align*}
y &= \frac{SV}{1 + A\beta} = 9.998 \times 10^{-4}
\end{align*}
\]

D) We can substitute \( A(s) = \frac{A_0 s_1}{s + s_1} \) directly into the expression for \( T \) from Part A:

\[
T = \frac{A(s)}{1 + A(s)\beta} = \frac{A_0 s_1}{s + s_1} \frac{1}{1 + \beta A_0 s_1} \frac{1}{s + s_1 + \beta A_0 s_1} \frac{A_0 s_1}{s + (1 + A_0 \beta) s_1}
\]

\[
T \approx \frac{A_0 s_1}{s + A_0 \beta s_1}
\]

This is the transfer function of a lowpass filter with a single pole at \( \omega = A_0 \beta s_1 \).
PROBLEM 10.2

These are all ideal opamps, therefore we make the following assumptions:

- $i_{in+} = i_{in-} = 0$
- $v_+ = v_-$
- $v_0$ is whatever it has to be to guarantee the other assumptions

A) $v_+ = 0$ implies that $v_- = 0$

All of the current, $i$, flowing through $R_1$, must flow through $R_2$, the input current, $i_{in+}$, is zero.

Therefore: $I = \frac{v_1}{R_1} - \frac{v_-}{R_2} = \frac{v_- - v_0}{R_2}$  ($v_+ = 0$)

\[
\frac{v_1}{R_1} = \frac{v_-}{R_2} \\
v_0 = \left(\frac{-R_2}{R_1}\right) v_1
\]

B) We can find $v_+$ using the voltage divider relationship:

$v_+ = v_2 \left[ \frac{R_2}{R_1 + R_2} \right]$  (and we know that $v_- = v_+$)

Now we use the relationship from Part A:

$\frac{v_1}{R_1} - \frac{v_-}{R_2} = \frac{v_- - v_0}{R_2}$

solving for $v_0$ and collecting terms gives

$v_0 = \left[ \frac{R_1 + R_2}{R_1} \right] v_- - \frac{R_2}{R_1} v_1$

substituting for $v_-$, we get

$v_0 = \left[ \frac{R_1 + R_2}{R_1} \right] \left[ \frac{R_2}{R_1 + R_2} \right] v_2 - R_2 \frac{v_1}{R_1}$  or  \[
v_0 = \frac{R_2}{R_1} \left[ v_2 - v_1 \right]
\]
c) Again, we can solve directly for \( V_+ \) (and therefore \( V_- \)).

\( V_+ \) is halfway between \( V_1 \) and \( V_2 \):

\[
V_+ = \frac{V_1 + V_2}{2} = V_-
\]

To find \( V_0 \), we write

\[
\frac{V_3 - V_-}{R} = \frac{V_- - V_0}{R} \Rightarrow V_0 = -\left(\frac{V_3 - 2V_-}{R}\right)
\]

\[
= 2V_- - V_3
\]

Substituting for \( V_- \) gives

\[
V_0 = 2\left(\frac{V_1 + V_2}{2}\right) - V_3 \Rightarrow V_0 = V_1 + V_2 - V_3
\]

**Problem 10.3**

For all of these opamp problems, we first solve directly for \( V_+ \) or \( V_- \) and assume the other one is the same.

Then we use the fact that the input currents are zero to write an expression relating \( V_0 \) to the input voltages and \( V_+ \) and \( V_- \).

Finally, we substitute for \( V_+ \) and \( V_- \) and solve for \( V_0 \).

\[
V_0 = 0 = V_- \\
I_D = \frac{K}{2} (V_{gs} - V_+)^2 = \frac{K}{2} \left(\frac{V_1 - V_2}{2}\right)^2 = V_2^0 \\
V_0 = -\frac{K}{2} \left[ V_1 - V_+ \right]^2 = -0.25 \left[ V_1 - \frac{1}{2} \right]^2
\]

This expression is only valid if \( V_1 > 1V \). If \( V_1 < 1V \), there is no current flowing and \( V_0 \) must be zero.

\[
V_0 = \begin{cases} 
-\left(\frac{V_1 - 1}{2}\right)^2 & V_1 > 1V \\
0 & V_1 < 1V 
\end{cases}
\]
PROBLEM 10.4

A) This is an inverting amplifier, similar to problem 10.2 part A. The only difference is that $R_2$ has been replaced by a complex impedance, $\frac{1}{Cs}$.

Therefore, $\frac{V_o}{V_i} = A(s) = -\frac{1}{Cs} \frac{1}{R}$

$A(s) = \frac{-1}{RCs}$ which is an integrator. (think of Laplace

Because the opamp is ideal, this expression holds for all frequencies: $0 \leq \omega \leq \infty$

But in the real world, the opamp has a finite gain, $A_0$, at low frequencies as shown below:

\[ A_0 \]

\[ \text{frequency response} \]

At some frequency $\omega = \omega_L$, the integrator runs out of "headroom". It needs more gain than the opamp can provide. So the $A(s) = \frac{1}{RCs}$ is only valid for $s >> \omega_L$.

\[ \text{desired integrator frequency response} = \frac{1}{RCs} \]

\[ \omega_L \]

\[ s_1 \]
B) This is a non-inverting amplifier topology. We can write $A(s)$ directly, in a way similar to Part A:

$$A(s) = \frac{V_o}{V_i} = 1 + \frac{R_{\text{in}}}{Ls}$$

where $Ls/\frac{1}{Cs} = \frac{L/C}{Ls + \frac{1}{Cs}} = \frac{Ls}{Ls^2 + 1}$

$$A(s) = 1 + \frac{R_{\text{in}}}{Ls} = 1 + \frac{RLCs^2 + R}{Ls} = \frac{Ls + RLCs^2 + R}{Ls}$$

$$A(s) = \frac{s^2 + \frac{1}{RC}s + \frac{1}{LC}}{s}$$

assuming $R$ is large enough, $A(s) \approx RC\left(\frac{s^2 + LC}{s}\right)$

see attached Bode Plot.

PROBLEM 10.5

We know that the left most opamp subcircuit is an inverter with a gain of 1. Therefore, its output is just $-V_1$. This is the input to the second subcircuit.

We know that this signal must be negative, because it's given that $V_1 > 0$.

The second subcircuit is an inverter with a non-linear element in the feedback path. $i_D$ must flow through $R_2$, therefore we can write:

$$i_D = K_2 (V_{gs} - V_T)^2 = \frac{V_1}{R_2}$$

Substituting ($V_2 = V_{gs}$) and ($V_T = 0$) and ($V = 0$) we get

$$\frac{K_2}{2} (V_2)^2 = \frac{V_1}{R_2}$$

Solving for $V_2$ and substituting, we get

$$V_2 = \sqrt{\left(\frac{2}{K_2 R_2}\right)V_1} = \sqrt{\frac{2(V_1)}{0.001 \times 1000}} = \sqrt{20(V_1)} = V_2$$
PROBLEM 10.6

For Parts A and B, we solve in similar fashion to Problems 10.5 and 10.3...

A) we can write

\[ \frac{V_i - V^2_0}{R} = i \approx I_s (e^{-\frac{V_0}{V_{th}}}) ; \quad V_{th} = \frac{K_T}{q} \]

\[ \ln \left( \frac{V_i}{R I_s} \right) = -\frac{V_0}{V_{th}} \Rightarrow V_0 = -\frac{K_T}{q} \ln \left( \frac{V_i}{R I_s} \right) \]

current can only flow through the diode if

\[ V_0 \text{ is negative} \]

B) \[ i \approx I_s e^{-\frac{V_i}{V_{th}}} = \frac{V_0}{R} \quad ; \quad V_{th} = \frac{K_T}{q} \]

\[ V_0 = R I_s e^{-\frac{V_i}{q / K_T}} \]

\[ V_0 \text{ is positive} \]

C) Here is a straightforward circuit, using the ideas that

- \[ \ln(AB) = \ln A + \ln B \]
- \[ e^{\ln x} = x \]

The detailed analysis is left to the reader.

(see the next page)
\[ U_1 = -\frac{kT}{q} \ln \left( \frac{y}{RI_s} \right) \]
\[ U_2 = -y = |y| \]
\[ U_3 = -\frac{kT}{q} \ln \left( \frac{|y|}{RI_s} \right) \]
\[ U_4 = U_1 + U_3 = -\frac{kT}{q} \left[ \ln \left( \frac{x}{RI_s} \right) + \ln \left( \frac{|y|}{RI_s} \right) \right] = -\frac{kT}{q} \ln \left( \frac{x|y|}{(RI_s)^2} \right) \]
\[ U_0 = RI_s e^{-\frac{kT}{q} U_4} = RI_s e^{\ln \left( \frac{x|y|}{(RI_s)^2} \right)} = \frac{RI_s x|y|}{(RI_s)^2} \]

\[ U_0 = \left( \frac{1}{RI_s} \right) x(|y|) \]

By reversing the direction of diode \( D_2 \), the inverter can be omitted and \( y < 0 \) can be connected directly:

\[ y < 0 \quad \stackrel{R}{\longrightarrow} \quad U_3 = \frac{kT}{q} \ln \left( \frac{|y|}{RI_s} \right) \]

Then \( U_4 \) can be computed using a difference amplifier:

\[ U_4 = U_1 - U_3 = -\frac{kT}{q} \left[ \ln \left( \frac{x}{RI_s} \right) + \ln \left( \frac{|y|}{RI_s} \right) \right] \]

This gives the same final result,
\[ U_0 = \left( \frac{1}{RI_s} \right) x(|y|) \] but with only 4 opamps.
Problem 10.4, Part B (L=1, C=1, R=10^{12})

Magnitude (dB)

Phase (deg)

Frequency (rad/sec)