Exercise 5.1: This problem studies the MOSFET amplifier shown below. Assuming that the MOSFET operates in its saturation region, defined by $i_D = 0.5K(v_{GS} - V_T)^2$ and $0 \leq v_{GS} - V_T \leq v_{DS}$, determine $v_{OUT}$ as a function of $v_{IN}$. Also, determine the range of $v_{IN}$, and the corresponding range of $v_{OUT}$, over which the MOSFET operates in its saturation region.
Problem 5.1: Both amplifiers shown below are constructed with a (non-quadratic) MOSFET having the \( v_{DS} - i_D \) characteristics shown graphically below. The characteristics are plotted for \( v_{GS} = 0.5 \) V, 0.55 V, 0.6 V, 0.65 V, 0.7 V, 0.8 V ... 1.5 V. For \( v_{GS} < 0.5 \) V, the MOSFET is cut off. Note that the last page of this problem set contains a larger graph of the MOSFET characteristics. It may be turned in with your problem set solutions.

![MOSFET Characteristics](image)

(A) Consider Amplifier A. Using graphical analysis, plot \( v_{OUT} \) as a function of \( v_{GS} \) for \( 0 \) V \( \leq v_{GS} \leq 1.5 \) V.

(B) Using your results from Part (A) determine the range of \( v_{GS} \), and the corresponding range of \( v_{OUT} \), over which the small-signal voltage gain of Amplifier A from \( v_{GS} \) to \( v_{OUT} \) is reasonably
constant at its largest value; use “engineering judgment”. (It is over this range that the amplifier provides its largest voltage gain with little distortion.) What is the corresponding small-signal voltage gain? Also, what is the small-signal voltage gain near \( v_{GS} = 0 \)? (Your answer to the last question explains why small signals cannot be input directly at \( v_{GS} \) if they are to be amplified.)

(C) Consider Amplifier B. This amplifier builds on Amplifier A, and uses the resistor network comprising \( R_1 \) and \( R_2 \) to bias \( v_{GS} \) and \( v_{OUT} \). In this way, the amplifier will be designed to amplify \( v_{IN} \) when it is centered around 0 V. Determine \( v_{GS} \) in terms of \( R_1, R_2, v_{IN} \) and the power supply voltage of 5 V.

(D) Let \( R_1 = 10 \, \text{k}\Omega \). Using your results from Parts (A), (B) and (C), determine \( R_2 \) so that the performance of Amplifier B satisfies the following two criteria.

For \( v_{IN} = 0 \) V, resistors \( R_1 \) and \( R_2 \) establish the bias value of \( v_{OUT} \). As \( v_{IN} \) varies around 0 V, \( v_{OUT} \) varies around its bias value. With this in mind, the first criterion is that \( v_{OUT} \) must be allowed to vary within \( \pm 1 \) V of its bias value while remaining within the range identified in Part (B). That is, Amplifier B must be designed to provide voltage gain with little distortion over a 2-V range of \( v_{OUT} \). Note that the range for \( v_{OUT} \) found in Part (B) is wider than 2 V and so this criterion does not uniquely specify \( R_2 \).

The second criterion is that the small-signal voltage gain of Amplifier B from \( v_{IN} \) to \( v_{OUT} \) should be maximized when \( v_{OUT} \) is within the range established by the first criterion. With this additional criterion, \( R_2 \) is uniquely specified.

(E) Given your design of \( R_1 \) and \( R_2 \) found in Part (D), what is the small-signal voltage gain of Amplifier B for values of \( v_{IN} \) near 0 V?

(F) The small-signal voltage gain found in Part (E) should be smaller than that found in Part (B). Why?
Problem 5.2: This problem continues to study the two-stage amplifier studied first in Problem 4.3. In this problem, let $v_{\text{IN}} = V_{\text{IN}} + v_{\text{in}}$ and $v_{\text{OUT}} = V_{\text{OUT}} + v_{\text{out}}$, where $V_{\text{IN}}$ and $V_{\text{OUT}}$ are the large-signal components of $v_{\text{IN}}$ and $v_{\text{OUT}}$, respectively, and $v_{\text{in}}$ and $v_{\text{out}}$ are the small-signal components of $v_{\text{IN}}$ and $v_{\text{OUT}}$, respectively.

(A) Assume that both MOSFETs are biased so that they operate in their saturation regions. Develop a small-signal circuit model for the amplifier that can be used to determine $v_{\text{out}}$ as a function of $v_{\text{in}}$. In doing so, assume that $V_{\text{IN}}$ defines the operating point around which the small-signal model is constructed, and evaluate all small-signal model parameters in terms of $V_{\text{IN}}$ as necessary.

(B) Use the small-signal model to determine $v_{\text{out}}$ as a function of $v_{\text{in}}$.

(C) Compare the small-signal voltage gain found in Part (B), defined as $v_{\text{out}}/v_{\text{in}}$, to that found in Part (F) of Problem 4.3. Explain any differences.

(D) Determine the small-signal Thevenin equivalent of the amplifier when it is viewed through its output port.

Problem 5.3: Consider again the amplifier described in Exercise 5.1. In this problem, let $v_{\text{IN}} = V_{\text{IN}} + v_{\text{in}}$ and $v_{\text{OUT}} = V_{\text{OUT}} + v_{\text{out}}$, where $V_{\text{IN}}$ and $V_{\text{OUT}}$ are the large-signal components of $v_{\text{IN}}$ and $v_{\text{OUT}}$, respectively, and $v_{\text{in}}$ and $v_{\text{out}}$ are the small-signal components of $v_{\text{IN}}$ and $v_{\text{OUT}}$, respectively.

(A) Using your result from Exercise 5.1, determine the small-signal voltage gain of the amplifier as a function of the input bias voltage $v_{\text{IN}}$. That is, determine $v_{\text{out}}/v_{\text{in}} = dv_{\text{OUT}}/dv_{\text{IN}}$ evaluated at $V_{\text{IN}}$.

(B) Again assume that the MOSFET is biased so that it operates in its saturation region. Develop a small-signal circuit model for the amplifier that can be used to determine $v_{\text{out}}$ as a function of $v_{\text{in}}$. In doing so, assume that $V_{\text{IN}}$ defines the operating point around which the small-signal model is constructed, and evaluate all small-signal model parameters in terms of $V_{\text{IN}}$ as necessary.

(C) Use the small-signal model to determine the small-signal voltage gain $v_{\text{out}}/v_{\text{in}}$. Compare this small-signal gain to that found in Part (A) and explain any differences.

(D) Determine the small-signal Thevenin equivalent of the amplifier when it is viewed through its output port.
MOSFET Characteristics

\[ v_{GS} = 0.5 \text{ V} \]
\[ v_{GS} = 0.55 \text{ V} \]
\[ v_{GS} = 0.6 \text{ V} \]
\[ v_{GS} = 0.65 \text{ V} \]
\[ v_{GS} = 0.7 \text{ V} \]
\[ v_{GS} = 0.8 \text{ V} \]
\[ v_{GS} = 0.85 \text{ V} \]
\[ v_{GS} = 0.9 \text{ V} \]
\[ v_{GS} = 1.0 \text{ V} \]
\[ v_{GS} = 1.1 \text{ V} \]

\[ i_D \text{ [mA]} \]
\[ v_{DS} \text{ [V]} \]