Enter all your work and your answers directly in the spaces provided on the printed pages. Answers must be derived or explained, not simply written down. The quiz is closed book, but calculators are allowed.

Indicate units on all numerical answers.

The quiz contains 6 pages including the cover sheet. Make sure that your quiz contains all 6 pages and that you hand in all 6 pages.

Grade: Problem 1
Problem 2
Problem 3
Total
Problem 1.

In the circuit below, conductance values are given for the resistors. The unit for conductance is Siemens (S), which is the same as reciprocal Ohms.

![Circuit Diagram]

Fill in the missing terms in the following set of node equations, written for the node voltages $e_1$ and $e_2$:

\[
\begin{align*}
\text{e}_1 \text{ Node:} & \quad 3S[e_1 - 6V] - 12A + 5S[e_1 - e_2] + 10A + 4S[e_1] = 0 \\
\text{Simplified:} & \quad 12S[e_1] - 5S[e_2] = 2A + 18\frac{V}{A} = 20\ A \\
\text{e}_2 \text{ Node:} & \quad 5S[e_2 - e_1] + 1S[e_2 - 6V] + [e_2] 2S = 0 \\
\text{Simplified:} & \quad 8S[e_2] - 5S[e_1] = 6A
\end{align*}
\]

\[
\begin{align*}
\frac{12S}{e_1} + \frac{-5S}{e_2} &= 20\ A \\
\frac{-5S}{e_1} + \frac{8S}{e_2} &= 6A
\end{align*}
\]
Problem 2.

Consider the following circuit with multiple sources:

![Circuit Diagram]

a) Find the value of $i_x$ when the 48 V voltage source acts alone, i.e., when all other sources are set to zero.

\[
\begin{align*}
R_{eq} & \Rightarrow \frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{12} \\
& = \frac{3}{12} = \frac{1}{4} \\
R_{eq} & = 4 \Omega
\end{align*}
\]

\[
V = \frac{4}{4+12} \cdot 48 = 12 \text{ V}
\]

\[
i_x = \frac{12 \text{ V}}{6 \Omega} = 2 \text{ A}
\]
Problem 2. (Continued)

![Circuit Diagram]

Note: This is the same circuit as on the previous page. It is reproduced here for your convenience.

b) Find the value of $i_x$ when the 24 V voltage source acts alone, i.e., when all other sources are set to zero.

c) Find the value of $i_x$ when the 48 V voltage source and the 24 V voltage source act together, with all other sources set to zero.

Reminder: Indicate units on all numerical values.

\[ b/12 \Rightarrow 4 \Omega \]

\[ 24v \quad \frac{4}{4+12} \]

\[ i_x = \frac{-4}{6} = -1A \]

\[ i_x(a) + i_x(b) = \]

\[ i_p(c) \quad \text{Superposition} \]
Problem 3.

In the circuit shown below in Figure 3a, the nonlinear device has the $I_D - V_D$ relation shown in Figure 3c on the following page. Part a) of this problem asks you to find the operating point of the device established by the 4 mA current source in Figure 3a, while part b) asks you to find the small signal response due the small signal source $i_s$ indicated in Figure 3b.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3a.png}
\caption{Figure 3a}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3b.png}
\caption{Figure 3b}
\end{figure}

a) Find the operating point of the nonlinear device, i.e., $I_D$ and $V_D$, in the circuit shown in Figure 3a.

b) An additional small signal source $i_s$ is added to the circuit as shown in Figure 3b. Determine $v_d$ the small signal response of the nonlinear device’s voltage due to $i_s$. Assume the units of $i_s$ are mA.

\begin{align*}
(a) & \\
\beta_{TH} &= 2k \Omega \\
I_{sc} &= 2 mA \\
4V &= i_d \cdot 2k \Omega + V_D \\
i_D &= \frac{4}{2k} = 2 mA \\
V_D &= 4V
\end{align*}

\begin{align*}
\therefore \text{op. pt.} & : I_D = 1.15 mA \\
& V_D = 1.75V
\end{align*}

\begin{align*}
(b) & \\
\frac{\partial \tilde{e}_d}{\partial \tilde{v}_d} = \frac{\tilde{v}_d}{\tilde{v}_d} = 2.5 \\
\text{or} & \\
r_d &= \frac{1}{\frac{\partial \tilde{e}_d}{\partial \tilde{v}_d}} = 2 k \Omega \\
\frac{\partial i_s}{\partial v_d} &= \frac{v_d}{\beta_{TH} + r_H} \\
\therefore \tilde{v}_d &= \frac{r_d \cdot \tilde{v}_d + \tilde{i}_s}{r_d + r_H}
\end{align*}

\begin{align*}
(a) & \\
I_D &= 1.15 mA \\
V_D &= 1.75V
\end{align*}
Figure 3c

v-i relation of the nonlinear device embedded in the circuits of Figures 3a and 3b.
Problem 2 (in AF1051C)

a. Thevenize the circuit around NL device.

\[ R_{TH} = 2k\Omega \]

Next calculate \( V_{TH} \) and \( I_{SC} \).

Given our circuit, \( I_{SC} \) is easier to determine as follows:

\[ I_{SC} = 2mA \]

\[ V_{TH} = I_{SC} \cdot R_{TH} = 4V \]

Now,

\[ V_{TH} = i_d R_{TH} + V_D \]

\[ 4V = i_d \cdot 2k\Omega + V_D \]

\[ \begin{cases} i_d = 2mA, V_D = 0 \\ i_d = 0, V_D = 4V \end{cases} \]

Op. Pt.:

\[ I_D = 1.15mA \]

\[ V_D = 1.75V \]
PART (b).

Now we introduce, small-signal, $i_S$ and want to know its effect on $n_D$.

Proceed as follows:

1. Find linear model for NL device that relates $i_D$ to $n_D$.
2. Use this model to characterize behavior of $n_D$ in the presence of input $i_S$.
3. Before doing anything, turn off large signal sources!

Around our op. point from PART (A), our NL-device looks linear:

$$\frac{di_D(n_D)}{dn_D} \bigg|_{V_B=1.75} = \frac{(2.3 - 1.15) \text{mA}}{(6 - 1.75) \text{V}} \approx 0.5 \text{mA/V}$$

Note: In a small-signal sense, NL device looks like a resistor with value

$$R_D = \frac{1}{\frac{di_D(n_D)}{dn_D} \bigg|_{V_B=1.75}} = \frac{2k\Omega}{1}$$

Use this to find effect of $i_S$ on $n_D$.

For Norton Eq. etc.,

\[ i_S = \frac{1}{2} \]

\[ \text{Current Divider:} \]

\[ U_D = \frac{R_D \cdot (500 \text{mA})}{R_{TH} + R_D} \text{ in mV} \]

or \[ (0.5k\Omega) i_S \text{ in volt} \]