6.02 Spring 2009
Lecture #12

- Frequency Division Multiplexing
- Why Complex Exponentials
- Frequency Response and Filters
- Zeros and Poles
New Problem - Resource Sharing

- Frequency Division Multiplexing Strategy
  - Represent each channel with a different frequency
    - For LTI systems, frequencies do not mix
      \[ x[n] = A_1 e^{j\Omega_1 n} + \ldots + A_K e^{j\Omega_K n} \]
    - Now need to separate the different frequencies
      - Use Filters to separate Y in to different channels
      - LTI systems with specific frequency responses
Eternal Complex Exponentials

\[ x[n] = e^{0.4jn} = \cos 0.4n + j \sin 0.4n \]
Frequency Response

- From convolution

\[ y[n] = \sum_{m=-\infty}^{\infty} h[m] e^{j\Omega(n-m)} \]

Reorganizing

\[ y[n] = \left( \sum_{m=-\infty}^{\infty} h[m] e^{-j\Omega m} \right) e^{j\Omega n} \]

A complex number if the sum converges

\[ y[n] = H(e^{j\Omega}) e^{j\Omega n} \]

\[ H(e^{j\Omega}), -\pi < \Omega \leq \pi, \text{ is the frequency response} \]
Recall Channel Unit Sample Responses

Slow Channel

Fast Channel
Magnitude of Frequency Response

Slow Channel

\[ |H(e^{j\Omega})| \]

Fast Channel

\[ |H(e^{j\Omega})| \]
Response to Cosine starting at zero

\[ x[n] = \cos(\Omega n)u[n] \quad \Omega = \left\{ \frac{\pi}{10}, \frac{2\pi}{10}, \frac{3\pi}{10} \right\} \]
Summary and Larger Picture

- **Frequency Division Multiplexing**
  - K channel’s each using a different frequency
    \[ z_k = e^{j\Omega_k}, \quad 0 \leq \Omega_k \leq \pi \]

- **Filtering**
  - Design \( |H(e^{j\Omega})| \)
  - Use Zeros to eliminate undesired frequencies
    - Must be in complex-conjugate pairs \( e^{j\Omega_k}, e^{-j\Omega_k} \)
  - Use Poles to magnify desired frequency
    - Magnitude < 1, and conjugate pairs \( re^{j\Omega_k}, re^{-j\Omega_k} \quad 0 < r < 1 \)

- **After Spring Break**
  - Encoding Information using different frequencies
    - What happens when we use
    \[ x[n] = A_1[n]e^{j\Omega_1n} + A_2[n]e^{j\Omega_2n} \]
    - Do the modulated complex exponentials still stay separated?