Using DFT pictures for Modulation
Examples of Plotting Spectrum

Lecture #15
6.02 Spring 2009

Digital Communication
Introduction to ECC II

SysteM
Recall Problem

\[ s_1[n] \rightarrow \times \rightarrow s_2[n] \cos(\theta_1 n) \]

What does this signal look like?

\[ s_2[n] \rightarrow \times \rightarrow s_2[n] \cos(-\theta_2 n) \]

\[ \cos(\theta_2 n) \]

If \( \Omega_1 \gg \text{band width of } s_1 \),
Reminder about DFT
Plot Many ways

$S[k]$ vs. $k$

$\frac{2\pi}{N} \cdot k$

$-2\pi < k < 2\pi$

Sampling frequency

$S[k]$

$F_s = 8000$ samples/sec

$8000 \cdot \frac{\pi}{2 \pi} = f_k$

$\frac{F_s}{2}$

Relating $k$ to $f_k$: $F_s \frac{2\pi}{N} \cdot k = \frac{F_s \pi}{2 \pi} = f_k$
$s[k] \text{ vs } k (73,313 \text{ samples})$
$I_C (S/k) \text{ vs } \omega$

$K [S \text{ vs } \Omega]$
Filtered $S[k]$ vs frequency (8K samp/sec)
Filterred S[k] vs k (73, 313 Samples)
Filtered S[K] vs Omega
010010 Bit sequence with slow rise
1001010 slow bit sequence (no stems)
\[ u_n \cos 0.5 \pi n = [u] z \]
\[ u \sin ([\gamma] \lambda) w \]

\[ u \sin ([\gamma] \lambda) \Re \phi \]

\[ u \cos (u \cos 0.5 \pi \cos [u] s) = [u] h \]
Problem Receiver does not know the phase
Addendum to Lec 15

The Algebra of Modulation

\( N \) assumed odd

Suppose \( \{ c[n] \} = \frac{L}{R} \sum_{k=-L}^{L} e^{j \frac{2\pi k n}{R}} \quad L \ll R \quad R = \frac{N-1}{2} \)

(L for limited)

Modulate by \( \cos \left( \frac{2\pi}{N} M n \right) \)

Assume \( M + L \leq R \)

\[ w[n] = s[n] \cos \left( \frac{2\pi}{N} M n \right) \]

\( \uparrow \)

Modulated Signal

\[ s[n] \quad \cos \left( \frac{2\pi}{N} M n \right) \]

\[ \sum_{k=-R}^{R} w[k] e^{j \frac{2\pi k n}{N}} = \left( \sum_{k=-L}^{L} s[k] e^{j \frac{2\pi k n}{N}} \right) \frac{1}{2} e^{j \frac{2\pi M n}{N}} + \frac{1}{2} e^{-j \frac{2\pi M n}{N}} \]

Determining \( w[k] \)

\[ \sum_{k=-R}^{R} w[k] e^{-j \frac{2\pi k n}{N}} = \]

\[ \sum_{k=-L}^{L} s[k] e^{j \frac{2\pi k n}{N}} + \sum_{k=-L}^{L} \frac{1}{2} s[k] e^{j \frac{2\pi (k+M) n}{N}} + \sum_{k=-L}^{L} \frac{1}{2} s[k] e^{j \frac{2\pi (k-M) n}{N}} \]
\[
\sum_{k=-B}^{B} W[k] e^{-j \frac{2\pi}{N} k \theta} = \\
\sum_{k=-L+M}^{L+M} \frac{1}{2} S[k-M] e^{-j \frac{2\pi}{N} k \theta} + \\
\sum_{k=-L-M}^{L-M} \frac{1}{2} S[k+M] e^{-j \frac{2\pi}{N} k \theta}
\]

Now Assume \( M > L \)

\( M - L > 0 \quad L - M < 0 \)

\[ W[k] = S[k-M] \quad k > 0 \]

\[ W[k] = S[k+M] \quad k < 0 \]

\( W[k] \) is nonzero if

\[ M - L \leq k \leq M + L \quad \text{or} \quad -L - M \leq k \leq L - M \]
Computing $\sum[k]$ from $s[n] = e^{j\frac{2\pi}{N}kn}$

$$\sum[k] = \sum_{n=0}^{N-1} s[n] e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}kn} e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-k)n}$$

If $k - k \neq 0$  $$\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-k)n} = 0$$

(Summing exactly $k-k$ periods of a sine and a cosine = 0)

If $k - k = 0$  $$\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-k)n} = \sum_{n=0}^{N-1} 1 = N$$

$\Rightarrow \sum[k] = N$  $k = \ell$

$\sum[k] = 0$ otherwise
0 = \mathcal{U}_n \frac{N}{2} \mathcal{E} \left[ u \right] S \bigcap_{1 - N} = \left[ \mathcal{U} \right] S

I + X \mathcal{U} = N \bigcap_{\mathcal{U} \mathcal{U}} \frac{N}{2} \mathcal{E} \left[ \mathcal{U} \right] S \bigcap_{X} \frac{N}{I} = \left[ u \right] S

(\text{Reminder about DFT (Watch for N)})

Post Lecture Recitation Notes:

\[ \]
There is a relation between Fourier series index $k$, the sample rate $f_s$, and the frequency $\Omega_k$ associated with the Fourier frequency $\Omega_k$:

\[ \Omega_k = \frac{2\pi k}{f_s} \]

This is a relation between Fourier series index $k$, the frequency $\Omega_k$, and the sample rate $f_s$.

In the following modulation example, a signal with a cosine frequency and then demodulated, once with a cosine, then shifted by $\pi/4$ and then demodulated, $\text{DFT}$ is demodulated with a cosine at 10 times the lowest

\[ u[n] = 2.0 + 0.5 \cos 0.5 \sin 2\pi n \]

\[ \frac{1}{2} \text{I}_1 \]

\[ U_1 \]

\[ 1 \text{I}_1 = N \]
Note that

$$s[n] = 2.0 + 0.5 \cos \frac{n}{2} + 2.0 \sin \frac{2n}{2}$$

Plot of $s[n]$, $\text{Re}(s[k])$, $\text{Im}(s[k])$.
Plot of \( \text{SIN}[\text{COS}1\text{OWN}]\text{COS}1\text{OWN} \)
Plot of (sin[n]cos[10\pi n])sin[10\pi n]
Note

Plot of \( \cos(10\pi n + \pi/4) \)
Plot of $\sin[n \cos(10 \pi n + \pi/4)]$
Plot of $s[n][\cos(10\pi n + \pi/4)][\cos(10\pi n)]$
Plot of \( \sin [\mu(\cos(10\mu) + \pi/4) + \mu(10\mu)] \)