Problem 1.1

Which way are polarizers in polarized sunglasses oriented? Why?

Problem 1.2

A typical laser consists of a gain medium located inside a resonator formed by two highly reflecting mirrors. Consider a gas laser with the design shown, in which Brewster windows are employed to seal the laser tube containing the gas (gain medium). Assume the light generated by the gain medium (when appropriately stimulated) is unpolarized, and that one of the mirrors is partially transmissive as shown. Describe, using Fig. 1, the polarization properties of the output beam from such a laser. Ignore multiple reflections from the Brewster window surfaces.

![Figure 1: Diagram of laser with Brewster polarizing windows](image)

Problem 1.3

Using dipole radiation arguments in conjunction with the diagrams below, describe and explain the general state of polarization of sun light that is scattered from the clear overhead sky towards the observer in each of the following three cases: (i) at sunrise, (ii) noon, and (iii) sunset. That is, your answer will explain the mechanism which accounts for the partial polarization of sky light. Draw as many diagrams as you need for a clear explanation. Perform the experiment to verify your answers.

![Figure 2: Observer looking at overhead sky at (i) dawn, (ii) noon and (iii) dusk](image)
Consider an electromagnetic plane wave that is described by

\[ E(\mathbf{r}, t) = \Re \left\{ (\hat{x} + 2j\hat{y})E_0e^{j\left(\frac{2\pi}{1 \times 10^{-5}} z - \frac{2\pi}{1 \times 10^{-13}} t\right)} \right\} \]

where all units are in MKS. For the following questions, be thorough with your derivations! No credit be given for answers without explanation or calculation.

(a) What is the direction of propagation of the wave?
(b) What is the polarization state of the wave?
(c) What is the value of the wave-number, \( k \)?
(d) What is the temporal frequency, \( \nu \), of the wave?
(e) What is the index of refraction, \( n \), of the medium?
(f) What is the free-space wavelength, \( \lambda \), of the wave?
(g) Write the equation for the \( \mathbf{H} \)-field associated with this wave.
(h) What is the complex Poynting vector for the wave?
(i) Compute the intensity of the wave by time-averaging the square of \( E(\mathbf{r}, t) \)
(j) Recompute the intensity from the complex Poynting vector.

Problem 1.5

(a) Derive the Jones Matrix for a linear polarizer with its axis oriented at +45° away from the x-axis towards the y-axis.
(b) Derive the Jones Matrix for a quarter-wave plate with its fast axis along the x-axis.

An optical isolator is a sandwich of a linear polarizer and a quarter-wave plate oriented as in parts (a) and (b). An optical isolator is illuminated with light of the appropriate wavelength and a mirror is placed after the optical isolator such that the light makes a double-pass through the optical isolator and back towards the source.

(c) Assuming the light first enters the optical isolator through the polarizer, use Jones matrices to compute the percentage of light that goes back towards the source.
(d) Assuming the light first enters the optical isolator through the quarter-wave plate, use Jones matrices to compute the percentage of light that goes back towards the source.

Problem 1.6 - 6.637 only

As was shown in class: (i) no light is transmitted through a system consisting of two crossed polarizers (polarizer No. 1 and Polarizer No. 2 (analyzer)), and (ii) light is transmitted through the system when a third polarizer (polarizer No. 3) is placed between the first two such that its axis is not aligned with the axes of either of the first two polarizers.
(a) If the incoming light is polarized along the axis of the first polarizer, what percentage of the input light intensity is transmitted through the system when polarizer No. 3 is aligned with its axis at 45° to the axis of the first polarizer.

(b) If we were to uniformly stack a large number, N, of linear polarizers between the first and second polarizers such that the axis of each polarizer in the stack is rotated by \(90/(N + 1)\) degrees from the axis of its neighbor. How will this alter the intensity of the transmitted light?

(c) Calculate the output intensity in the limit where N approaches infinity.

**Problem 1.7 - 6.637 only**

The goal is to determine the polarization state, \(\hat{e}_{\text{out}}\), and the relative intensity, \(I_{\text{out}}\), of the output light in each of the four systems shown below. Derive your answers by writing the amplitude and the polarization state of the light on both sides of all the components.

Notes: (1) A polarization beam splitter (PBS) passes TM and reflects TE light. The operation of a normal beam splitter is polarization independent. (2) Ignore phase changes on reflection and transmission in this problem.

(e) Apply the Jones matrix methods in (a)-(d) to confirm your findings.