Problem 3.1

Two mutually coherent intersecting plane waves of equal amplitude $A$, wavelength $\lambda_1$ and separation angle $\theta$, are incident symmetrically on a planar slab of recording material of refractive index $n$ as shown.

![Figure 1](image_url)

(a) Write equations for the two waves inside the slab.

(b) Assuming the phase difference between the waves is zero at $(x = 0, z = 0)$, use your equations to derive expressions for the separation $\Lambda$ of the interference fringes in the material.

(c) Show via geometric construction of the fringes that your results are correct.
Problem 3.2

An infinitely wide plane wave beam of TM-polarized light ($\lambda = 500$ nm) is incident upon a thin soap film of refractive index $n = 1.3$ and thickness $d \, \mu$m at the Brewster angle. The wavelength of the light is $0.5 \, \mu$m

(a) What is this angle of incidence?

(b) What is the reflectivity, $R$, of the air-soap interface for this beam?

(c) Are there fringes in Region I arising from the reflected beam? Show your reasoning. If so, what is the interference condition for a bright maximum?

(d) Are there fringes in Region III arising from the transmitted beam? Show your reasoning. If so, what is the interference condition for a bright maximum?

Suppose the polarization of the beam, still incident at the Brewster angle, was changed to TE polarization:

(e) Give an expression for the reflectivity $R_{TE}$ of the air glass interface

(f) What is the minimum thickness $d$ of the soap film that gives a strong reflected beam at this angle of incidence (Show your reasoning)
Problem 3.3

The Young’s double slit experiment is modified, as shown below, by placing a thin parallel glass plate of thickness $d$ and refractive index $n$ over one of the slits. Here $a$ is the slit width in the $x$ direction and $b$ is the slit separation also in the $x$ direction. The system is excited with on-axis collimated laser light of wavelength $\lambda$. A lens of focal length $F$ is placed immediately behind the slits and a screen is placed a distance $F$ away from the lens, so that a set of well-defined interference fringes are visible on the screen. Ignore all phase changes from reflection and transmission in this problem.

(a) For the case where the glass plate is absent, derive an expression for the position, $X_{2m}$, of the $m$th maximum on the screen.

(b) Derive an expression for the spatial frequency shift $\Delta f = [(\sin \theta')/\lambda - (\sin \theta)/\lambda]$ that occurs when the glass slide is inserted in the position shown in the diagram. What important conclusion do you draw from your expression?

(c) Derive an expression for the lateral spatial shift $\Delta X_{2m} = X'_{2m} - X_{2m}$ on the screen as a result of inserting the glass slide.

(d) In which direction do the fringes shift?
Problem 3.4

Two lenses were found in the laboratory. One is plano-convex with a 50cm radius of curvature, and the other is plano-concave with an unknown radius of curvature. The refractive index of the lens material is \( n = 1.52 \) for both lenses. So you decided to set up the Newton’s rings interference experiment shown below in order to find the radius of curvature of the plano-concave lens. In the setup, the curved surfaces of both lenses are placed in contact and the system illuminated with collimated monochromatic light of wavelength 550nm. With the aid of a beamsplitter, you observe a set of circular fringes called ”Newton’s rings”. in particular you witness that the 20\(^{th}\) bright ring has a diameter of 12mm.

![Newton's rings setup](image)

Figure 2: Observing, by use of a beam splitter, Newton’s fringes formed between two abutted lenses.

(a) What is the radius of curvature, \( R_{\text{concave}} \), of the concave lens surface?

(b) What are the focal lengths, \( F_{\text{concave}} \) and \( F_{\text{convex}} \), of both lenses?