Problem 4.1

The figure below is the plot of a piecewise-continuous, one-dimensional crossection through a 2-D wavefront of wavelength $\lambda$. The normal to the wavefront segments all lie in the $x-z$ plane, and the wavefront is travelling nominally in the $+z$-direction. All segments of the 2-D wavefront are of the same area ($10\lambda \times 10\lambda$) and they all carry the same power density of $I$ Watt/m$^2$.

(a) Write a frequency-domain expression, $U(f_x, f_z)$, that describes the primary directions of power flow in this wavefront. (i.e., ignore any effective aperaturing and therefore diffraction effects)

(b) Sketch the spatial-frequency content of this wavefront on a graph with the co-ordinate system shown below.

(c) The wavefront passes through a lens of focal-length $F$ (not shown). Sketch the intensity pattern that would be seen on a screen placed in the back focal plane of the lens, and label the positions and the sizes of any critical features that will be present on the screen.
Problem 4.2

A 20mW CO$_2$ laser of wavelength $\lambda = 10.6 \ \mu m$ aboard an unmanned aerial vehicle (UAV) is pointed at a passenger plane flying at a distance of 9.75km from the UAV. The laser beam is 2mm in diameter immediately after exiting the laser. Assume there is no atmospheric turbulence.

(a) What is the spot size of the laser beam on the passenger plane?

(b) Assuming that all the light from the laser is distributed evenly within the spot on the airplane, what is the power density of the beam on the passenger plane?

(c) Can a passenger looking out of the window of the airplane at the UAV see the laser light? Why?

(d) Can you devise a simple, practical means (that is implementable aboard the UAV) that will increase the optical power density on the passenger plane?

Problem 4.3

A plane-wave of amplitude $A$ and wavelength $\lambda$ is incident at normal incidence in the $\hat{z}$-direction on the transmission object $U_a(x, y)$ described below.

$$
U_a(x, y) = \begin{cases} 
  e^{j\frac{\pi}{2}} & 0 < x < a \\
  e^{-j\frac{\pi}{2}} & -a < x < 0 \\
  0 & \text{elsewhere} 
\end{cases}
$$

(a) Draw a sketch of this object in the x-y plane

(b) Give an example of how you would fabricate such an object.

(c) Compute analytically the intensity of the Fraunhofer diffraction field owing to this object.

(d) Plot the Fraunhofer diffraction field of part (c) using your favorite software package.
Problem 4.4

A plane-wave of amplitude $A$ and wavelength $\lambda$ is incident at normal incidence in the $\hat{z}$-direction on the transmission object $U_b(x,y)$ described below.

$$U_b(x,y) = \begin{cases} 
1 & a_1 < x < a_2 \quad b_1 < y < b_2 \\
1 & -a_1 > x > -a_2 \quad b_1 < y < b_2 \\
1 & a_1 < x < a_2 \quad -b_1 > y > -b_2 \\
1 & -a_1 > x > -a_2 \quad -b_1 > y > -b_2 \\
0 & \text{elsewhere}
\end{cases}$$

$$a_1 \geq 0, \quad b_1 \geq 0, \quad a_2 > a_1, \quad b_2 > b_1$$

(a) for simplicity, let $a_0 = (a_1 + a_2)/2, \quad a = (a_2 - a_1), \quad b_0 = (b_1 + b_2)/2, \quad \text{and} \quad b = (b_2 - b_1)$, Draw a sketch of this object in the x-y plane.

(b) Compute analytically the intensity of the Fraunhofer diffraction field owing to this object.

(c) Plot (using your favorite software) the intensity of the Fraunhofer diffraction field of part (b). We are interested in the “overall” pattern, not exact numbers. Use reasonable values for $a$ and $b$ when plotting.

(d) Input the structure $\{U_b(x,y)\}$ into Matlab or Mathematica, and plot it’s Fourier transform (intensity - the magnitude of the Fourier transform squared). Compare your results with the analytical plots you got in part (c).

Hint: Use the `fft` command in Matlab to get the Fourier transform of the transmission function. Remember, Matlab’s plot command only takes real numbers as input, whereas the output of `fft` is complex. The best way to do this is to ‘draw’ the object with your favorite paint program, using the normalized values $a_1 = 1, \quad b_1 = 1, \quad a_2 = 2, \quad b_2 = 3$. Disregard the dependence of the diffraction pattern on the wavelength of incident light. Note, if using a paint program, that the object must be small with respect to the entire canvas on which it is drawn to get a well-defined transform (otherwise your output will look like a $\delta$-function – which is incorrect). You may use the code below to *assist* you (this code was written for Lab #3).

conv2fourier.m

```matlab
function result = conv2fourier(input); % create conv2fourier function
matrix = imread(input); % read input file as a 2-D array of values
figure; zoom on; % create a new figure window with zooming enabled
v = fftshift(abs(fft2((255 - double(matrix))))).^2; % do some touch-up work, and take the Fourier transform, so that we arrive at a easily viewable representation of the 2-D fft of the bitmap image
imagesc(v(900:1150,900:1150)); % plot the result as an image... the original for this example had a canvas of 2048 x 2048 pixels in size and a much smaller image centered around (x = 1024 pixels, y = 1024 pixels).
```

To use the code given here, at the Matlab prompt type:

```matlab
conv2fourier('smiley.bmp'); % take the bitmap image smiley.bmp and show its corrected Fourier transform
```
Problem 4.5

For an N-slit grating of pitch Λ and slit-width \( a = \Lambda / 2 \), derive an expression for the angular width of the zero-order fringe when the grating is read out with light of wavelength \( \lambda \). Graphically plot the width of the zero-order fringe (with \( \lambda / \Lambda \) as a scale factor) from \( N = 1 \) to \( N = \infty \).