Problem 5.1

In the coordinate system below, the following definitions hold:

\[ \overline{\rho} = x\hat{x} + y\hat{y} \]
\[ \rho = x\hat{x} + y\hat{y} + z\hat{z} \]
\[ \overline{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z} \]

Show the direction of propagation, and algebraically derive, sketch and describe the shape of the wavefront associated with the following elementary unit amplitude waves:

(a) \( U(\rho) = e^{j\overline{k}\cdot\rho} \)
(b) \( U(\rho) = e^{jkz}e^{-jk\rho^2/\rho} \)
(c) \( U(\rho) = e^{jkz}(x^2+y^2) \)
(d) \( U(\rho) = e^{jk\sqrt{(x-x_0)^2+(y-y_0)^2}} \)
(e) \( U(\rho) = e^{jk(x^2+y^2)^{1/2}} \)
(f) \( U(\rho) = e^{jk[z^2+(x-x_0)^2+(y-y_0)^2]^{1/2}} \)
Problem 5.2

(a) Does the writing geometry in Fig. 1 lead to a transmission or reflection hologram?

(b) Assume that the hologram shown in Figure 1 is a thin hologram and that it is read out with the conjugate beam of $A (A^*)$ as defined in the notes. Draw a diagram of the readout geometry.

(c) Derive the field expressions (amplitude and phase) of each exiting beam.

(d) What is the direction (in terms of $\alpha$) of each exiting beam?

(e) Complete the diagram in (b) to show each entering and exiting beam with its appropriate field equation; also label the real and virtual images, if they exist.

![Figure 1: Hologram written with reference beam, $A$, and object beam, $U_0(x, y)$](#)

Problem 5.3

Thick reflection holograms are used for decorative purposes on credit cards, and are also worn (usually around the neck) as decorative jewelry. Large reflection holograms can also be found as art media in museums (e.g., MIT Museum).

A “thick” reflection hologram of an object $U_0(x, y)$ is made in a recording medium of refractive index $n$ using the setup shown below. The writing light has wavelength $\lambda_w$. The plane-wave reference beam has amplitude $A_r$ and is incident from below in the $x - z$ plane at an angle $\alpha$.

![Figure 2: Hologram written with reference beam, $A$, and object beam, $U_0(x, y)$](#)
(a) For the case where the recording medium is thick, draw and describe the location of the images when the hologram is read out with the conjugate of the reference beam.

(b) Next, the thick hologram is read out in an optimal way (Bragg matched) with a plane wave of wavelength $3\lambda_w/2$ also incident from above on the front side.

(1) What is the optimal angle, $\psi$, between the $z$-axis and this readout beam?

(2) Describe the characteristics of the images produced, and draw a diagram of this optical readout configuration showing the location of the images.

(c) Using the results from part (b), describe the output images that are obtained when this hologram is read out with collimated white light incident from above on the front side.

**Problem 5.4**

(a) Our goal is to make a thick white light reflection hologram using the recording setup shown below. Assume the reference beam is a spherical wave. The hologram is to be read out with a center reconstruction wavelength $\lambda_r = 532\,nm$. However, owing to the recording material development process, we know that the recording emulsion will shrink to 80% of its original size. What must be the recording (write) wavelength $\lambda_w$ to get a faithful reproduction of the object with the 532 nm spherical readout wave whose origin is at the same location as that of the reference beam?

![Diagram of recording setup](image)

(b) In the case where the same wavelength is used for writing and readout, assuming again that we have 20% emulsion shrinkage, how will the readout geometry have to change to view the hologram? Draw one or two diagrams to help clarify your explanation, and in these, show clearly the location of the readout source and location of the image thus formed.
Problem 5.5

A hologram is made with waves derived from an on-axis object \( U_o(\rho) \) located in the \( z = 0 \) plane and a point source of amplitude \( A \) located on the \( x \)-axis at a distance \( a \) above the principal axis as shown.

(a) Write an expression for the object wave, \( U_o(\rho, z_0) \), at the recording medium.

(b) Write an expression for the spherical reference wave, \( U_r(\rho, z_0) \), at the recording medium.

(c) Assuming the recording medium has a transmitted amplitude response that is proportional to exposure, write an expression for the amplitude transmittance of the resulting hologram.

(d) The hologram is to be read out with the conjugate of the reference wave. Write an expression for the conjugate reference wave at the \( z = z_0 \) plane and draw a diagram to show the wavefronts of the conjugate reference wave reading out the hologram.

(e) Compute the output terms corresponding to the waves generated by the hologram when read out with the conjugate reference beam, and illustrate these on the diagram you made in part (d).