Problem 2.1T

For all real values of parameter $a$ find the coefficients $T, R, L$ of a feedback transformation of coordinates

$$x = Tz, \quad u = Rv + Lz$$

which transforms system

$$\dot{x} = \begin{bmatrix} a & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} u$$

into the canonical form.

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1Version of February 21, 2001
Problem 2.2T

Give upper bounds (best of what you can find) of $L_2$ gains of the following systems (with input $f$ and output $y$), where $a$ is a real parameter.

(a) $y(t) = \cos(a f(t)) - 1$.

(b) $y(t) = \sin^2(t)f(t) - f(t - |a|)$.

(c) $\ddot{y}(t) + a\dot{y}(t) + y(t) = f(t)$.

(d) $y(t) = \int_{t-|a|}^{t} f(\tau)d\tau$.

(e) $y(t) = |f(t)||a|$.

(f) $y(t) = \max_{t \geq \tau \geq t-|a|} f(\tau)$.

(g) $y(t) = \int_{n-|a|}^{n} f(\tau)d\tau$ for $n \leq t < n+1$.

Problem 2.3T

The time varying system model

$$\ddot{x}(t) = (a + b \cos(rt))x(t) + f(t)$$

(with input $f$ and output $x$) can be represented in the form of a feedback interconnection of LTI subsystem

$$\ddot{x}(t) = ax(t) + b(\dot{w}(t) + w(t)) + f(t), \quad z(t) = x(t) + \dot{x}(t),$$

and a time-varying subsystem $w = \Delta z$. Give an upper bound $\gamma$ for the $L_2$ gain of $\Delta$. (The upper bound should be such that $\gamma \to 0$ as $r \to \infty$.)

Problem 2.4E

System $S$ with input $f$ and output $y$ is defined by

$$y(t) = \int_{t-1}^{t} f(\tau)d\tau.$$ 

Find a first order transfer function model $H(s)$ such that the “error” system $\Delta = S - H$ has $L_2$ gain which is as small as you can make it.