Problem Set 5

This is a completely “theoretical” homework, designed to help with the preparation for Quiz 1. The problem set consists of a large number of problems which are “small” in the following sense: if the concept behind the question is well understood, a simple solution can be obtained with no or very few calculations.

The homework is due on April 4, in the format of a hardcopy or an electronic file.

Problem 5.1T

For all values of the real parameter \( a \in \mathbb{R} \) find the H-Infinity norm of the transfer matrix

\[
G(s) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{s}{s^2 + as + 1} \end{bmatrix}.
\]

Problem 5.2T

For all values of parameter \( T \geq 0 \) find the H2 norm of

\[
G(s) = \frac{e^{-Ts}}{s} - 1.
\]

\(^1\)Version of March 21, 2001
Problem 5.3T

Give an example of a canonical output feedback design setup with scalar state, control, noise, sensor and cost variables, such that there is a single control singularity at $\omega = 0$ and a single sensor singularity at $\omega = \infty$.

Problem 5.4T

Give Q-parameterization for the set of all closed loop transfer functions from $w$ to $z$ which can be obtained by using a stabilizing proper LTI controller in $C(s)$ the system on Figure 5.1 with

$$P(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{(s+1)^2} \end{bmatrix}.$$

![Figure 5.1: Setup for Problems 5.4T and 5.8T](image)

Problem 5.5T

Matrix $K$ is such that $A + BK$ has a single eigenvalue (of multiplicity 3), where

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}.$$

Find all possible values of the eigenvalue.

Problem 5.6T

For all values of $a \in \mathbb{R}$ find the minimum (or infimum) in the abstract H2 optimization problem

$$\dot{x} = -x + u, \quad x(0) = 1, \quad x(\infty) = 0,$$

$$\int_0^\infty (u^2 + 2axu)dt \to \inf.$$
**Problem 5.7TT**

Write down a system of LMIs with respect to the scalar parameters $d_1, d_2$ and, possibly, some other parameters, which has a solution if and only if

$$d_0 + \delta_1 \omega^2 + \omega^6 > 0 \quad \forall \omega \in \mathbb{R}.$$  

**Problem 5.8T**

For the setup of Figure 5.1 with

$$P(s) = (s - 1)^2 / (s + 1)^3$$

give a good lower bound for the H-Infinity norm of the closed loop complementary sensitivity transfer function $T : w \to z$ provided that $C$ is a stabilizing controller and

$$|T(j\omega) - 1| < 0.1 \quad \forall \omega \in [0, 10].$$