Problem 1 of 4 (25 points)

Answers without work shown will not be given any credit.

At the base of a vertical cliff, a model rocket, starting from rest, is launched upwards at $t = 0$ with a time-varying acceleration given by $a_y(t) = A - Bt$, where $A$ and $B$ are positive constants. The rocket rises to a maximum height and then descends until it returns to the ground. Also at $t = 0$, a small stone is released from rest from the top of the cliff at a height $h$ directly above the rocket. (This height $h$ is higher than the maximum height reached by the rocket.) The stone hits the rocket at the instant when the rocket reaches its maximum height. The gravitational acceleration is $g$ downward. You may neglect air resistance.

a) How long after the rocket was launched did the stone hit the rocket? Express your answer in terms of the constants $A$, $B$, and $g$, as needed.

b) Determine an expression for the height $h$ from which the stone was dropped in terms of the constants $A$, $B$, and $g$, as needed.
\( y_1 = A - B \tau \)
\[
\nu_{y_1}(t_f) - \nu_{y_1}(t_0) = \int_{t_0}^{t_f} (A - B \tau') d\tau' = A t_f - \frac{1}{2} B t_f^2 + 4 \rho t_f
\]
\( y_1(t_f) = 0 \Rightarrow A t_f - \frac{1}{2} B t_f^2 = 0 \Rightarrow t_f = \frac{2A}{B} + 3 \rho t_f
\]
\[
y_1(t_f) = y_1(t_0) = \int_{t_0}^{t_f} (A t' - \frac{B t'^2}{2}) d\tau' = \frac{A t_f^2 - B t_f^3}{6} + 5 \rho t_f
\]
\( y_2 = -g \)
\[
\nu_{y_2}(t_f) = \nu_{y_2}(t_0) = \int_{t_0}^{t_f} -g \tau' = -g t_f
\]
\[
y_2(t_f) - y_2(t_0) = \int_{t_0}^{t_f} -g \tau' d\tau' = -\frac{g t_f^2}{2}
\]
\( y_2(t_0) = h \)
\( y_2(t_f) = h - \frac{1}{2} g t_f^2 + 5 \rho t_f
\)
\( y_1(t_f) = y_2(t_f) \Rightarrow h = \frac{A t_f^2 - B t_f^3}{2} - \frac{g t_f^2}{2}
\]
\( h = \frac{1}{2} (g + A) t_f^2 - \frac{B t_f^3}{6}
\]
\( h = \frac{1}{2} (g + A) \left( \frac{2A}{B} \right)^2 - \frac{B}{6} \left( \frac{2A}{B} \right)^3
\]
\( = \frac{2}{B^2} \left( g + A \right) A^2 - \frac{4}{3} \frac{A^3}{B^2}
\]
\( = \frac{A^2}{B^2} \left( 2g + \frac{2}{3} A \right) + 3 \rho t_f
\)
Problem 2 of 4 (25 points)

Answers without work shown will not be given any credit.

A block slides on a surface inclined at an angle $\theta$ to the horizontal. The mass of the block is $M$. The coefficient of kinetic friction between the block and the surface is $\mu_k$. At $t=0$, the block has an initial speed $v_0$ pointing up the inclined plane. The block eventually stops and then slides back down the plane.

![Diagram of a block on an inclined plane with coefficient of kinetic friction $\mu_k$ and angle $\theta$.]

a) Find a vector expression for the acceleration of the block when it is sliding up the inclined plane. Be sure to define your coordinate system clearly.

b) Find a vector expression for the acceleration of the block when it is sliding down the inclined plane. Be sure to define your coordinate system clearly.

c) Determine how long it takes for the block to slide up the inclined plane, stop, and then slide down the incline plane until the instant it is moving with speed $v_0$ pointing down the inclined plane. (You may assume that the angle $\theta$ is large enough that the block does not get stuck at its maximum height.)
\( a) + 9 \)

\[ \begin{align*}
\text{1 pt: } & \quad f_k + mg\sin\theta = ma_{x1} \\
\text{1 pt: } & \quad N - mg\cos\theta = 0 \\
\text{1 pt: } & \quad f_k = \mu_k N \Rightarrow f_k = \mu_k mg\cos\theta \\
\text{1 pt: } & \quad a_{x1} = gs\sin\theta + \mu_k g\cos\theta \\
\text{1 pt: } & \quad a_{y1} = 0
\end{align*} \]

\( 1) + 9 \)

\[ \begin{align*}
\text{1 pt: } & \quad mg\sin\theta - f_k = ma_{x2} \\
\text{1 pt: } & \quad N - mg\cos\theta = 0 \\
\text{1 pt: } & \quad f_k = \mu_k N \Rightarrow f_k = \mu_k mg\cos\theta \\
\text{1 pt: } & \quad a_{x2} = gs\sin\theta - \mu_k g\cos\theta \\
\text{1 pt: } & \quad a_{y2} = 0
\end{align*} \]
\[ v_x(t) = v_{x_0} + a_{x_1} \cdot t \quad 1 \text{ pt} \]
\[ -v_0 + a_{x_1} \cdot t_1 = 0 \quad 1 \text{ pt} \]
\[ \Rightarrow t_1 = \frac{v_0}{a_{x_1}} = \frac{v_0}{g \sin \theta + \mu_k g \cos \theta} \quad 1 \text{ pt} \]
\[ v_0 (t_2) = a_{x_2} \cdot t_2 \quad 1 \text{ pt} \]
\[ t_2 = \frac{v_0}{a_{x_2}} = \frac{v_0}{g \sin \theta - \mu_k g \cos \theta} \quad 1 \text{ pt} \]
\[ t_{tot} = t_1 + t_2 = \frac{v_0}{g} \left( \begin{array}{cc} 1 & -1 \\ \sin \theta + \mu_k g \cos \theta & \sin \theta - \mu_k g \cos \theta \end{array} \right) \quad 1 \text{ pt} \]
Problem 3 of 4 (25 points): Blocks-pulley system

Answers without work shown will not be given any credit.

Block 1 of mass $m_1$ rests on a horizontal frictionless table. Block 2 of mass $m_2$ rests on top of block 1. Block 1 is connected by a massless inextensible string wrapped around a massless ideal pulley to block 3 of mass $m_3$, as shown in the figure above. The coefficient of static friction between blocks 1 and 2 is $\mu_s$. The gravitational acceleration is $g$ downward.

When the system is released from rest, block 1 accelerates to the right and block 2 does not slip relative to block 1.

a) Draw free-body (force) diagrams for each of the three blocks. Note: You should clearly define your choice of coordinate system and unit vectors for each object.

b) Find an expression for the acceleration block 3 in terms of $m_1$, $m_2$, $m_3$, $\mu_s$, and $g$ as needed.
\[ f_3 = m_1 a_1 \]

\[ f_3 = m_2 a_2 \]

\[ m_3 g - T = m_3 a_3 \]

\[ a_1 = a_2 = a_3 = a \]

\[ a = \frac{m_3 g}{m_1 + m_2 + m_3} \]
Problem 4 of 4: (25 points) Concept Questions (Parts A through E)

Part A (5 points): Three identical railroad cars, labeled 1, 2, and 3 in the figure below, are pulled with a force of magnitude $F$ by a locomotive. You may ignore any effects of friction.

![Diagram of railroad cars being pulled by a force](image)

Denote the magnitude of the force on car 2 due to car 1 by $F_{12}$ and the magnitude of the force on car 2 due to car 3 by $F_{32}$. Which of the following answers is true?

1. $F_{12} = F_{32} = F/3$.
2. $F_{12} = F_{32} = F$.
3. $F/3 = F_{12} < F_{32} = 2F/3$.
4. $F = F_{12} < F_{32} = 2F$.
5. None of the above.

Answer: 3
An object is moving along the $x$-axis. At $t = 0$, it is located at the origin. The graph above is a plot of the $x$-component of velocity versus time, $v_x(t)$ vs. $t$. Which of the following is a correct description of the motion? (More than one answer may be a correct description of the motion.)

1. During the interval $0 < t < 3 \, \text{s}$, the object reaches a maximum distance in the positive $x$-direction from the origin and then goes backwards.

2. During the interval $0 < t < 3 \, \text{s}$, the $x$-components of the object’s position and velocity are positive, and the $x$-component of the acceleration changes sign.

3. The $x$-component of the acceleration is zero when the object reaches its maximum speed and during the interval $2 \, \text{s} < t < 3 \, \text{s}$.

4. During the interval $0 < t < 3 \, \text{s}$, the $x$-component of the object’s position is positive, but the $x$-components of its velocity and acceleration change sign.

5. During the interval $0 < t < 3 \, \text{s}$, the $x$-components of the object’s position, velocity and acceleration all change sign.

**Answer(s): 2,3**
Part C (5 points):

A juggler, standing at rest on the ground, is spinning a plate at a constant angular speed \( \omega \). An ant crawls out of a hole in the center of the plate and starts to walk radially outwards along a line drawn on the plate, at a constant speed \( v_0 = \left( \frac{R}{2} \right) \omega \) relative to the rotating plate. When the ant reaches the point A on the plate, a distance \( \frac{R}{2} \) from the center, which of the following arrows best describes the direction of the velocity of the ant according to the juggler?

Answer: 2
Part D (5 points):

A system of two pulleys, two blocks, and inextensible strings are shown in the figure below.

Choose positive \( y \)-direction downwards for both blocks one and two. What is the relationship between the \( y \)-components of the accelerations of blocks 1 and 2 due to the constraints imposed by the system of strings and pulleys?

1. \( a_{y1} = a_{y2} / 2 \).
2. \( a_{y1} = -a_{y2} / 2 \).
3. \( a_{y1} = a_{y2} \).
4. \( a_{y1} = -a_{y2} \).
5. \( a_{y1} = 2a_{y2} \).
6. \( a_{y1} = -2a_{y2} \).

Answer: 2
Part E (5 points): Starting at the same instant, two objects A and B are pushed on a frictionless surface from a starting line to a finish line with equal constant forces. Object B is four times as massive as object A. Both objects are initially at rest.

1. Object A has four times the acceleration as object B and reaches the finish line in half the time as object B.

2. Object B has four times the acceleration as object A and reaches the finish line in half the time as object A.

3. Both objects reach the finish line at the same time with the same speed.

4. Object A has four times the acceleration as object B and reaches the finish line in one quarter the time as object B.

5. Object B has four times the acceleration as object A and reaches the finish line in one quarter the time as object A.

6. Object A has twice the acceleration as object B and reaches the finish line in half the time as object B.

7. Object B has twice the acceleration as object A and reaches the finish line in half the time as object A.

8. Not enough information is given to decide.

Answer: 1