Newton’s Second Law: Force, Mass, Acceleration: \[ \vec{F} = m \vec{a} \]

Newton’s Third Law: \[ \vec{F}_{1,2} = -\vec{F}_{2,1} \]

Center of Mass: \[ \vec{R}_{cm} = \frac{1}{m_{\text{total}}} \sum_{i=1}^{i=N} m_i \vec{r}_i \rightarrow \frac{1}{m_{\text{total}}} \int \text{body} dm \vec{r} \]

Velocity of Center of Mass: \[ \vec{V}_{cm} = \frac{1}{m_{\text{total}}} \sum_{i=1}^{i=N} m_i \vec{v}_i \rightarrow \frac{1}{m_{\text{total}}} \int \text{body} dm \vec{v} \]

Momentum: \[ \vec{p} = m\vec{v} \text{, } \vec{p}_{\text{sys}} = \sum_{i=1}^{i=N} m_i \vec{v}_i \]

Newton’s Second Law \[ \vec{F}_{\text{ext}} = \frac{d\vec{p}_{\text{sys}}}{dt} \]

Impulse: \[ \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta \vec{p} \]

Kinetic Energy: \[ K = \frac{1}{2} mv^2 ; \Delta K \equiv \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \]

Work- Kinetic Energy: \[ W = \int_A^B \vec{F} \cdot d\vec{r} \quad W = \Delta K \]

Potential Energy: \[ \Delta U \equiv U(B) - U(A) \equiv -W_c = -\int_A^B \vec{F}_c \cdot d\vec{r} \]

Potential Energy Functions with Zero Points:

Constant Gravity: \[ U^g(y) = mgy \text{ with } U^g(y = 0) = 0 \]

Inverse Square Gravity: \[ U^G(r) = -\frac{Gm_1m_2}{r} \text{ with } U^G(r = \infty) = 0 \]

Springs: \[ U^s(x) = \frac{1}{2} kx^2 \text{ with } U^s(x = 0) = 0 \]

Work- Mechanical Energy: \[ W_{\text{nc}} = \Delta K + \Delta U_{\text{total}} = \Delta E_{\text{mech}} = \left( K_f + U_{f}^{\text{total}} \right) - \left( K_0 + U_0^{\text{total}} \right) \]
## Exam 2

**Physics 8.01**

**Fall Term 2013**

### Name (IN CAPITALS)

_______________________________

### Student ID Number

_______________________________

### Table Number

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### Section: Check the correct section

- [ ] L01 Dourmashkin MW 9-11 F9
- [ ] L02 Vuletic MW 11-1 F11
- [ ] L03 Frebel MW 1-3 F1
- [ ] L04 Belcher MW 3-5 F4
- [ ] L05 England TR 9-11 F10
- [ ] L06 Chakrabarty TR 11-1 F12
- [ ] L07 Mavalvala TR 2-4 F3

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Grader Initial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>(25 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Problem 2</td>
<td>(25 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Problem 3</td>
<td>(25 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Problem 4</td>
<td>(25 points)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>(100 points)</td>
<td></td>
</tr>
</tbody>
</table>
Problem 1 of 4 (25 points)

Answers without work shown will not be given any credit.

![Diagram showing a block on a frictionless track and a semi-circular frictionless track, with labels for mass, normal force, and friction coefficient.]

A block of mass $m$ is released from rest at a height $3R$ above the ground and slides down a frictionless vertical track. When it reaches a height $R$ above the ground it enters a semi-circular frictionless track of radius $R$. At the bottom of the semi-circular track it enters a horizontal track starting at $x=0$. The coefficient of kinetic friction along the horizontal track is non-uniform and varies according to $\mu(x) = bx$, where $b$ is a positive constant. The block slides and comes to rest at $x=d$. The magnitude of the gravitational acceleration is $g$.

a) What is the normal force of the semi-circular track on the block when the block makes an angle $\theta$ as shown in the figure above? Express your answer in terms of $m$, $R$, $\theta$, and $g$ as needed.

b) Calculate the distance $d$ that the block slides along the horizontal track before coming to a rest. Express your answer in terms of $m$, $R$, $b$, and $g$ as needed.
Problem 2 of 4 (25 points)

Answers without work shown will not be given any credit.

Two astronauts are playing catch in a zero gravitational field. Astronaut 1 of mass $m_1$ is initially moving to the right with speed $v_1$. Astronaut 2 of mass $m_2$ is initially moving to the right with speed $v_2 > v_1$. Astronaut 1 throws a ball of mass $m$ with speed $u$ relative to herself in a direction opposite to her motion. Astronaut 2 catches the ball. The final speed of astronaut 1 is $v_{f,1}$ and the final speed of astronaut 2 is $v_{f,2}$.

![Diagrams before, during, and after the catch]

a) What is the speed $v_{f,1}$ of astronaut 1 after throwing the ball? Express your answer in terms of $m$, $m_1$, $m_2$, $u$, and $v_1$, as needed.

b) What is the required speed $u$ of the ball (relative to astronaut 1) such that the final speed of both astronauts are equal $v_{f,1} = v_{f,2}$? Express your answer in terms of $m$, $m_1$, $m_2$, $v_1$, and $v_2$, as needed.
Problem 3 of 4 (25 points)

Answers without work shown will not be given any credit.

A two-stage rocket before ignition

A two-stage rocket in a zero gravitational field starts from rest and burns fuel. The fuel is ejected at a speed \( u \) relative to the rocket. After all the fuel has been burned, explosive bolts release the first stage and push it backwards with a speed \( v_1 \) relative to the second stage. The mass of the first stage before any fuel is burned is \( m_1 \equiv m_0 + m_f \), where \( m_f \) is the mass of the fuel. The mass of the second stage is \( m_2 \). The total mass of the rocket before any fuel is burnt is \( m_1 + m_2 \). The goal of this problem is to find the speed of the second stage after the separation.

a) When the rocket is traveling at speed \( v \), derive a relation between the differential of the speed of the rocket \( dv \), and the differential of the mass of the rocket, \( dm_r \), the mass of the rocket \( m_r \), and the speed \( u \) relative to the rocket of the ejected fuel. You must show your work in order to receive credit.

b) What is the speed \( v_f \) of the rocket immediately after all the fuel has been burned but before the second stage is released? Express your answers in terms of \( u \), \( m_f \), \( m_0 \), and \( m_2 \) as needed.

c) What is the speed \( v_2 \) of the second stage immediately after it has been released? Express your answers in terms of \( v_1 \), \( v_f \), \( m_f \), \( m_2 \), and \( m_0 \) as needed.
Problem 4 of 4 (25 points): Concept Questions (Parts A through E)

Part A (5 points)

The potential energy function $U(x) = (1/2)kx^2$ for a particle with total mechanical energy $E$ is shown below.

The particle was released moving in the positive $x$-direction at the point $x = x_1$ at $t = 0$. The $x$-component of the velocity of the particle at $t = 0$ is

1) $v_x(t = 0) = 0$,

2) $v_x(t = 0) = \pm \sqrt{\frac{2E}{m}}$,

3) $v_x(t = 0) = + \sqrt{\frac{2}{m} \left( E - \frac{1}{2}kx_1^2 \right)}$,

4) $v_x(t = 0) = \sqrt{\frac{kx_1^2}{m}}$,

5) None of the above.

Answer: _________________________
Part B (5 points)

Consider two carts, of masses $m$ and $4m$, at rest on an air track. If you first push both carts with an equal force $F$ an equal distance $d$, then the change in momentum of the light cart is

1) one-fourth  
2) one-half  
3) equal to  
4) twice  
5) four times

the momentum of the heavy cart.

Answer: _________________________
Part C (5 points)

A worker is unloading the last car (mass $m_c$) from a barge (mass $m_b$) tied to a pier by a short taut cable. Starting at rest, he drives the car with a constant acceleration until it reaches the end of the barge at time $t_0$. What is the tension in the cable during the interval $0 < t < t_0$?

1) $m_c a_c$.

2) $(m_c + m_b) a_c$.

3) $\frac{m_c}{m_c + m_b} a_c$.

4) $\frac{m_b}{m_c + m_b} a_c$.

5) None of the above.

Answer: _________________________
Part D (5 points)

Suppose water is leaking out through the hole of the bottom of a cart that is moving with speed $v$ on a frictionless surface. You may ignore all the effects of air resistance. While the water is leaking out, the speed of the cart

1) increases but is independent of the rate that the water is leaking out of the cart.

2) increases but depends on the rate that the water is leaking out of the cart.

3) does not change.

4) decreases but is independent of the rate that the water is leaking out of the cart.

5) decreases but depends on the rate that the water is leaking out of the cart.

Answer: _________________________
Part E (5 points)

A point-like object of mass $m$ is attached to one end of a massless string that is fixed at the point $S$. The object moves in a vertical circle of radius $r$ centered at $S$ with constant angular speed $\omega$ in a zero gravitational field. Which of the vectors 1-6 best describes the velocity, acceleration, and net force acting on the object at the moment illustrated in the figure above?

Answer: _________________________