A uniform cylinder of radius $R$ and mass $M$ is initially given a horizontal speed $v_0$ and a counterclockwise angular speed $\omega_0 = v_0 / 2R$ on the frictionless part of a horizontal surface. Beyond point $A$, the surface changes to a rough area so that the coefficient of kinetic friction to the right of point $A$ is $\mu_k$. After the cylinder passes $A$, it will at first slip on the rough plane, but it will eventually roll without slipping. The moment of inertia of the cylinder about its center is $I_{cm} = (1/2)MR^2$.

a) What is the speed of the center of mass of the cylinder when the cylinder starts to roll without slipping?

b) What is the ratio of the final kinetic energy (when it is rolling without slipping) to the initial kinetic energy?

c) How far past point $A$ does the center of mass travel before the cylinder starts to roll without slipping?
\[ \tau_s = 0 \Rightarrow \ell_{s,i} = L_{s,f} \]

\[ L_{s,i} = I_{s,cm} \cdot M \cdot \omega_{cm,i} + \ell_{cm,i} = R \cdot M \cdot V_0 \cdot \omega_0 \cdot k \]

\[ L_{s,f} = (R \cdot M \cdot V_f + I_{cm} \cdot \omega_f) \cdot k \]

\[ R \cdot M \cdot V_0 - I_{cm} \cdot \omega_0 = R \cdot M \cdot V_f + I_{cm} \cdot \omega_f \]

\[ I_{cm} = \frac{1}{2} \cdot m \cdot R^2 \quad \omega_f = \frac{V_f}{R}, \quad \omega_0 = \frac{V_0}{2 \cdot R} \]

\[ R \cdot M \cdot V_0 - \frac{1}{2} \cdot m \cdot R^2 \cdot V_0 = R \cdot M \cdot V_f + \frac{1}{2} \cdot m \cdot R^2 \cdot \frac{V_f}{R} \]

\[ \frac{3}{4} \cdot m \cdot V_0 \cdot R = \frac{3}{2} \cdot m \cdot R \cdot V_f \Rightarrow \]

\[ V_f = \frac{1}{2} \cdot V_0 \]

\[ k_f = \frac{1}{2} \cdot m \cdot v_f^2 + \frac{1}{2} \cdot I_{cm} \cdot \omega_f^2 = \frac{1}{2} \cdot m \cdot v_f^2 + \frac{1}{2} \cdot \left( \frac{1}{2} \cdot m \cdot R^2 \right) \cdot \frac{V_f^2}{R^2} \]

\[ = \frac{3}{4} \cdot m \cdot V_f = \frac{3}{16} \cdot m \cdot V_0^2 \]

\[ k_i = \frac{1}{2} \cdot m \cdot v_0^2 + \frac{1}{2} \cdot I_{cm} \cdot \omega_0^2 = \frac{1}{2} \cdot m \cdot v_0^2 + \frac{1}{2} \cdot \left( \frac{1}{2} \cdot m \cdot R^2 \right) \cdot \frac{V_0^2}{R^2} \]

\[ = \frac{1}{16} \cdot m \cdot V_0^2 \Rightarrow \frac{k_f}{k_i} = \frac{3}{16} \cdot m \cdot V_0^2 = \frac{3}{3} \]

\[ W_{nc} = \Delta K = k_f - k_i = \frac{3}{16} \cdot m \cdot V_0^2 - \frac{9}{16} \cdot m \cdot V_0^2 = -\frac{3}{8} \cdot m \cdot V_0^2 \]

\[ W_{nc} = -f_k d = -A_{kmg} d = -\frac{3}{8} \cdot m \cdot V_0^2 \]

\[ l = \frac{3}{8} \cdot \frac{V_0^2}{A_{kmg}} \]
Problem 2 of 4 (25 points)

Answers without work shown will not be given any credit.

Two objects of equal mass $M$ are suspended by paper tape from a massless and frictionless pulley, as shown. The paper tape has negligible weight. Object A is a simple weight with the tape attached. Object B is a uniform cylinder of radius $R$ around which the paper tape is wrapped. The tape does not slip with respect to the cylinder. The moment of inertia of cylinder B about its center of mass is $I_{cm} = (1/2)MR^2$. The system is released from rest. Find the tension in the tape.
massless pulley

\[ R(T_B - T_A) = I_{cm} \alpha \]

\[ \pm_{cm} = 0 \implies T = T_B = T_A \]

\[ m_A g - T = m_A a_A \quad \text{(5)} \]

\[ m_B g - T = m_B a_B \quad \text{(5)} \]

\[ m = m_A = m_B \implies mg - T = ma_A \implies a_A = a_B = a \]

\[ mg - T = ma_B \]

\[ \vec{\tau}_{cm} = \pm_{cm} \vec{\alpha} \quad \text{(6)} \]

\[ K_T = I_{cm} \alpha \]
\[ l = l_c + R(\theta - \theta_c) \]
\[ l = y_A + y_B + \pi R^2 \]
\[ y_A + y_B + \pi R^2 = l_c + R(\theta - \theta_c) \]
\[ q_A + q_B = R \alpha \]
\[ 2a = R \alpha \]

\[ mg - T = ma \quad (1) \]
\[ RT = \frac{1}{2} m R^2 \alpha \quad (2) \]
\[ 2a = R \alpha \quad (3) \]

\[ R(mg - ma) = \frac{1}{2} m R^2 \frac{2a}{R} \quad \Rightarrow \]
\[ 2ma = mg \quad \Rightarrow \quad a = \frac{g}{2} \]
\[ \alpha = \frac{2a}{R} = \frac{2g}{2R} = \frac{g}{R} \quad (3) \]
\[ T = \frac{1}{2} m R \alpha = \frac{1}{2} m \frac{R g}{R} = \frac{mg}{2} \quad (2) \]
Problem 3 of 4 (25 points)

Answers without work shown will not be given any credit.

Three point-like objects are located at the points A, B and C, as shown below. They have masses $M_A = M$, $M_B = M$, and $M_C = 2M$. The three objects are initially oriented along the $y$-axis and connected by rods of negligible mass, each of length $D$, forming a rigid body.

A fourth object of mass $M$ moving with velocity $v_0 \hat{i}$ collides and sticks to the object at point A. After the collision, the four masses and connecting rods move as a new rigid body. The $z$-axis points out of the page. For the following questions, neglect gravity and give all your answers in terms of $M$, $v_0$, and $D$, as needed.

a) After the collision, what is the direction and magnitude of the linear velocity of the center of mass of the new rigid body?

b) After the collision, what is the magnitude of the angular velocity of the new rigid body?

c) Immediately after the collision, what is the direction and magnitude of the velocity $\vec{v}_C$ of the mass located at the point C?
2) \( \vec{F}_{ext} = 0 \Rightarrow P_{x,i} = P_{x,f} \)

\( P_{x,i} = m v_0 \)

\( \Rightarrow v_0 \)

\( P_{x,i} = P_{x,f} \Rightarrow \)

\( m v_0 = 5m v_{cm,f} \Rightarrow \)

\( v_{cm,f} = \frac{1}{5} v_0 \)
(b) calculate about point B

\[ \vec{L}_{B,i} = \vec{L}_{B,f} \]

\[ \vec{L}_{B,i} = \vec{D} \vec{m} \vec{V}_O \hat{k} \]

\[ \vec{L}_{B,f} = \vec{I} \vec{W}_f \hat{k} \]

\[ I_B = (2m)_D^2 + (2m)_D^2 = 4mD^2 \]

\[ \vec{L}_{B,f} = 4mD^2 \vec{W}_f \hat{k} \]

\[ \vec{L}_{B,i} = \vec{L}_{B,f} \Rightarrow \vec{D} \vec{m} \vec{V}_O = 4mD^2 \vec{W}_f \Rightarrow \]

\[ \vec{W}_f = \frac{\vec{V}_O}{4D} \]

\[ (c) \]

\[ \vec{V}_c = \vec{V}_{cm} + \vec{V}_{rot} \]

\[ = \frac{\vec{V}_O}{5} \hat{i} - D \vec{W}_f \hat{i} \]

\[ \vec{V}_c = \left( \frac{\vec{V}_O}{5} - \frac{D \vec{V}_O}{4D} \right) \hat{i} \]

\[ \vec{V}_c = -\frac{1}{20} \vec{V}_O \hat{i} \]
Problem 4 of 4: (25 points) Concept Questions (Parts A through E)

Part A (5 points)

A disc on ice (frictionless) is pulled by a massless string wound around the edge of the disc. The string does not slip with respect to the disc. The string is pulled with force $F$. The disc has mass $m$, radius $R$, and moment of inertia $I_{cm} = (1/2)mR^2$ about its center of mass. Which of the following relations hold for the acceleration of the center-of-mass and the angular acceleration of the disc?

![overhead view of a disc with a string around it](image)

a) Circle the correct answer for the acceleration of the center of mass. (2 points)

1. $a_{cm} = F/m$. **correct**
2. $a_{cm} > F/m$.
3. $a_{cm} < F/m$.

b) Circle the correct answer for the angular acceleration of the disc. (3 points)

1. $\alpha_{cm} < FR/I_{cm}$.
2. $\alpha_{cm} = FR/I_{cm}$. **correct**
3. $\alpha_{cm} > FR/I_{cm}$
4. $\alpha_{cm} < I_{cm}/FR$.
5. $\alpha_{cm} = I_{cm}/FR$.
6. $\alpha_{cm} > I_{cm}/FR$. 
Part B (5 points):

A rigid hoop of radius $R$ and mass $m$ lies on a horizontal frictionless table and is pivoted at point $P$, as shown below. The hoop is free to swing around the pivot. A point-like object of the same mass $m$ is moving to the right (see figure) with speed $v_0$. It collides elastically with the hoop at the midpoint of the hoop. After the collision, the point-like object moves with an unknown speed $v_f$ to the left, and the hoop rotates counterclockwise about its pivot point with angular speed $\omega_f$.

After the collision, how does the magnitude of the angular momentum of the hoop about the pivot point $P$ compare to the angular momentum of the point-like object about the pivot point $P$?

(a) It is greater.

(b) It is equal.

(c) It is lesser.

(d) Not enough information is specified to decide which of the above three statements is correct.

Enter the letter of the correct statement on the answer line below.

Answer:  

\[ \text{a) } \]
Part C (5 points):

A woman, holding identical weights in each of her hands at her sides, spins on a rotating stool. (Assume that the stool has no frictional torque acting along its axis of rotation.) She then stretches her arms outward to each side. Which of the following statements is true?

a) Her angular momentum about a point on the axis of rotation decreases, and her rotational kinetic energy decreases.

b) Her angular momentum about a point on the axis of rotation remains the same, and her rotational kinetic energy decreases.

c) Her angular momentum about a point on the axis of rotation increases, and her rotational kinetic energy decreases.

d) Her angular momentum about a point on the axis of rotation decreases, and her rotational kinetic energy remains the same.

e) Her angular momentum about a point on the axis of rotation remains the same, and her rotational kinetic energy remains the same.

f) Her angular momentum about a point on the axis of rotation increases, and her rotational kinetic energy remains the same.

g) Her angular momentum about a point on the axis of rotation decreases, and her rotational kinetic energy increases.

h) Her angular momentum about a point on the axis of rotation remains the same, and her rotational kinetic energy increases.

i) Her angular momentum about a point on the axis of rotation increases, and her rotational kinetic energy increases.

Enter the letter of the correct statement on the answer line below.

Answer: b)
Part D (5 points):

Two cylinders at the top of an inclined plane (shown below) have the same size and mass. However, Cylinder A is a hollow metal pipe (a cylindrical shell), and Cylinder B is a solid wood cylinder. They are each released from rest at the same instant, and they both roll without slipping down the inclined plane. Which reaches the bottom first?

a) Cylinder A  
b) Cylinder B  
c) Both reach the bottom at the same time.

Enter the letter of the correct statement on the answer line below.

Answer: b)

\[ mg\dot{h} = \frac{1}{2} m v_r^2 + \frac{1}{2} I_{cm} \omega^2 \]

\[ v = R \omega \]

\[ mg\dot{h} = \frac{1}{2} (m + I_{cm}) \dot{r}^2 \]

\[ v = \sqrt{\frac{2mg\dot{h}}{m + I_{cm} R^2}} \]

\[ I_A > I_B \Rightarrow v_B < v_A \]
Part E (5 points):

A uniform rod of length $L$ and mass $m$ is in static equilibrium. It is hinged to a wall at one end, and suspended from the wall by a massless cable that is attached to the other end of the rod at an angle $\beta = 30^\circ$ with respect to the horizontal. A block of equal mass $m$ is resting on the rod a distance $3L/4$ from the hinge point. Let $T$ denote the magnitude of the tension in the cable, and let $F_y$ denote the $y$-component of the hinge force acting at the point where the rod is hinged to the wall. Note that $\sin(30^\circ) = 1/2$ and $\cos(30^\circ) = \sqrt{3}/2$. Which of the following statements is correct?

a) $T = \frac{5\sqrt{3}}{6}mg$ and $F_y = +\frac{3}{4}mg$.

b) $T = \frac{5\sqrt{3}}{6}mg$ and $F_y = -\frac{3}{4}mg$.

c) $T = \frac{5}{4}mg$ and $F_y = +\frac{3}{4}mg$.

d) $T = \frac{5}{4}mg$ and $F_y = -\frac{3}{4}mg$.

e) $T = \frac{5}{2}mg$ and $F_y = +\frac{3}{4}mg$.

f) $T = \frac{5}{2}mg$ and $F_y = -\frac{3}{4}mg$.

g) None of the above.

Enter the letter of the correct statement on the answer line below.

Answer: _____e)______________________
\[ mg \frac{L}{4} + mg \frac{3L}{4} - \frac{LT}{2} = 0 \]

\[ \frac{5}{4} mg = \frac{T}{2} \]

\[ T = \frac{5}{2} mg \]

\[ F_y + \frac{T}{2} - 2mg = 0 \]

\[ F_y = 2mg - \frac{T}{2} = 2mg - \frac{5}{4} mg \]

\[ = \frac{3}{4} mg \]