Final Exam: Equation Summary

Newton’s Second Law: Force, Mass, Acceleration: \( \vec{F} = m \vec{a} \)

Newton’s Third Law: \( \vec{F}_{1,2} = -\vec{F}_{2,1} \)

Center of Mass: \( \vec{R}_{\text{cm}} = \frac{1}{m_{\text{total}}} \sum_{i=1}^{N} m_i \vec{r}_i \rightarrow \frac{1}{m_{\text{total}}} \int \text{dm} \vec{r} \)

Velocity of Center of Mass: \( \vec{V}_{\text{cm}} = \frac{1}{m_{\text{total}}} \sum_{i=1}^{N} m_i \vec{v}_i \rightarrow \frac{1}{m_{\text{total}}} \int \text{dm} \vec{v} \)

Momentum: \( \vec{p} = m \vec{v} \), \( \vec{p}_{\text{sys}} = \sum_{i=1}^{N} m_i \vec{v}_i \)

Newton’s Second Law: \( \vec{F}_{\text{ext}} = \frac{d\vec{p}_{\text{sys}}}{dt} \)

Impulse: \( \vec{I} = \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta \vec{p} \)

Kinetic Energy: \( K = \frac{1}{2} m v^2 \), \( \Delta K \equiv \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \)

Work-Kinetic Energy: \( W = \int_{A}^{B} \vec{F} \cdot d\vec{r} \) \( W = \Delta K \)

Potential Energy: \( \Delta U \equiv U(B) - U(A) \equiv -W_c = -\int_{A}^{B} \vec{F}_c \cdot d\vec{r} \)

Potential Energy Functions with Zero Points:

Constant Gravity: \( U(y) = mgy \) with \( U(y_0 = 0) = 0 \).

Inverse Square Gravity: \( U_{\text{gravity}}(r) = -\frac{G m_1 m_2}{r} \) with \( U_{\text{gravity}}(r_0 = \infty) = 0 \).

Springs: \( U_{\text{spring}}(x) = \frac{1}{2} k x^2 \) with \( U_{\text{spring}}(x = 0) = 0 \).

Work-Mechanical Energy: \( W_{\text{nc}} = \Delta K + \Delta U_{\text{total}} = \Delta E_{\text{mech}} = \left( K_f + U_f^{\text{total}} \right) - \left( K_0 + U_0^{\text{total}} \right) \)
Moment of Inertia: 
\[ I_p = \int_{\text{body}} dm(r_\perp)^2 \]

Moment of inertia of uniform disk of mass \( M \) and radius \( R \) about axis passing through center of mass perpendicular to plane of disk: \( (1/2)MR^2 \)

Moment of inertia of uniform disk of mass \( M \) and radius \( R \) about axis passing through center of mass parallel to plane of disk: \( (1/4)MR^2 \)

Moment of inertia of uniform rod of mass \( M \) and length \( L \) about axis passing through center of mass perpendicular to rod: \( (1/12)ML^2 \)

Parallel Axis Theorem: 
\[ I_p = md^2 + I_{cm} \]

Torque about a point \( S \): 
\[ \tau_S = \mathbf{\hat{r}}_{S,F} \times \mathbf{F} \]

Angular Momentum (point particle) about a point \( S \): 
\[ \mathbf{L}_S = \mathbf{\hat{r}}_S \times m\mathbf{v} \]

Angular Impulse: 
\[ \int_{t_i}^{t_f} \tau_S \, dt = \mathbf{L}_{S,f} - \mathbf{L}_{S,i} \]

Fixed Axis Rotation (about z-axis):

Angular Velocity: 
\[ \mathbf{\dot{\omega}} = \omega \mathbf{\hat{k}} \]

Angular Acceleration: 
\[ \mathbf{\ddot{\alpha}} = \alpha \mathbf{\hat{k}} \]

Angular Momentum for fixed axis rotation (symmetric body): 
\[ \mathbf{L}_z = I_z \omega \mathbf{\hat{k}} \]

Torque and Angular momentum about point \( S \): 
\[ \tau_S^{\text{ext}} = \frac{d\mathbf{L}_S^{\text{sys}}}{dt} \]

Rotational Kinetic Energy about fixed point \( S \): 
\[ K_S^{\text{rot}} = \frac{1}{2} I_z \omega^2 \]

Rotation and Translation:

Angular Momentum about a point \( S \): 
\[ \mathbf{L}_S = \mathbf{L}_S^{\text{orbital}} + \mathbf{L}_S^{\text{spin}} = (\mathbf{\hat{r}}_{S,cm} \times m\mathbf{v}_{cm}) + \mathbf{L}_S^{\text{spin}} \]

Torque about a point: 
\[ \tau_S = \frac{d\mathbf{L}_S}{dt} \] (fixed point \( S \)), 
\[ \tau_{cm} = \frac{d\mathbf{L}_S^{\text{spin}}}{dt} \] (center of mass)

Kinetic Energy: 
\[ K = K_{trans} + K_{rot} = \frac{1}{2} m_{\text{total}} v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 \]