A crane is configured as below, with the beam (which we can define as massless, for simplicity) suspended at two points \( l_1 \) and \( l_2 \) by each end of a cable passing over a frictionless pulley. The two ends of the cable each make an angle \( \theta \) with the beam. A counterbalance object C with mass \( m_c \) is fixed at one end of the beam. A balance object B of mass \( m_b \) is attached to the beam and can move horizontally in order to maintain static equilibrium. The crane lifts an object A with mass \( m_A \) at a distance \( y \) from the counterbalance.

\[ \begin{aligned} &\text{a)} \text{ Draw a free body force diagram for the beam.} \\
&\text{b)} \text{ What is the tension in the cable that runs over the pulley, as a function of the masses of the hanging objects and the angle } \theta \text{ between the cable and the beam?} \\
&\text{c)} \text{ At what horizontal position should one put the balance object B such that the crane doesn’t tilt?} \\
\end{aligned} \]

\[ \begin{aligned} &\text{Solutions:} \\
&\text{a)} \text{ There are five forces, three downward tension forces } \vec{T}_A, \vec{T}_B, \text{ and } \vec{T}_C \text{ exerted by the rope connected the weights to the beam, and two tension forces } \vec{T}_1 \text{ and } \vec{T}_2 \text{ exerted by the cable wrapped around the pulley.} \\
\end{aligned} \]
b) The net horizontal force is zero and so both ropes make the same angle $\theta$, the tension in the ropes must be the equal, $T_1 = T_2$. The net upward force supplied by the cable is then $2T \sin \theta$. For equilibrium, this must be equal to the downward forces.

$$2T \sin \theta - F_A - F_B - F_C = 0.$$  \hspace{1cm} (1)

Because the hanging objects are at rest, by Newton’s Second Law $F_A = m_A g$, $F_B = m_B g$, and $F_C = m_C g$. Therefore the tension in the cable wrapped around the pulley is

$$T = \frac{(m_A + m_B + m_C)g}{2 \sin \theta}. \hspace{1cm} (2)$$

c) Although any point could be used for calculating torques, the distances are all measured from the left end of the beam in the above diagram, so using this point $S$ will simplify calculations. Also, the counterbalance force $F_C = m_C g$ exerts no torque about $S$. There are then four forces exerting torques.

![Diagram](image)

The torque about the point $S$ is then

$$\vec{\tau}_S = \vec{r}_{S1} \times \vec{T}_1 + \vec{r}_{S2} \times \vec{T}_2 + \vec{r}_{S,B} \times \vec{F}_B + \vec{r}_{S,C} \times \vec{F}_C$$

$$\vec{\theta} = \left( l_1 \hat{i} \times (T \cos \theta \hat{i} + T \sin \theta \hat{j}) \right) + \left( l_2 \hat{j} \times (T \cos \theta \hat{i} + T \sin \theta \hat{j}) \right)$$

$$\left( x \hat{i} \times m_B g \hat{j} \right) + \left( y \hat{j} \times m_A g \hat{j} \right)$$

$$\vec{\theta} = ((l_1 + l_2)T \sin \theta - (x m_B + y m_A)g) \hat{k}$$

$$\hspace{1cm} (3)$$

We can now substitute Eq. (2) into Eq. (3) and solve for $x$
\begin{equation}
0 = \frac{(l_1 + l_2)(m_A + m_B + m_C)g}{2} - (x m_B + y m_A)g \Rightarrow \\
x = \frac{1}{m_B} \left( \frac{(l_1 + l_2)(m_A + m_B + m_C)g}{2} - y m_A \right). \tag{4}
\end{equation}