WE_W11D2-2 Worked Example Descending and Ascending Yo-Yo Solution

A Yo-Yo of mass \( m \) has an axle of radius \( b \) and a spool of radius \( R \). It’s moment of inertia about the center of mass can be taken to be \( I = (1/2)mR^2 \) and the thickness of the string can be neglected. The Yo-Yo is released from rest. What is the acceleration of the Yo-Yo as it descends?

\[
\tau_{cm} = \mathbf{r}_{cm} \times \mathbf{T}.
\]  

\text{(1)}
About the center of mass, \( \mathbf{r}_{cm} = -b \mathbf{i} \) and \( \mathbf{T} = -T \mathbf{j} \), so the torque is

\[
\mathbf{\tau}_{cm} = \mathbf{r}_{cm} \times \mathbf{T} = -b \mathbf{i} \times -T \mathbf{j} = bT \mathbf{k} .
\]  

(2)

Applying Newton’s Second Law in the \( \mathbf{j} \)-direction,

\[
m g - T = ma_y
\]  

(3)

Applying the torque equation for the Yo-Yo:

\[
bT = I \alpha_z
\]  

(4)

where \( \alpha \) is its angular acceleration. The angular acceleration and the linear acceleration are related by the constraint condition

\[
a_y = b \alpha_z
\]  

(5)

Substituting Eq. (5) into Eq. (4) and then solve for tension yields

\[
T = I a_y / b^2
\]  

(6)

where \( b \) is the radius of the axle of the Yo-Yo. Now substitute Eq. (6) into Eq. (3) and solve for the acceleration

\[
a_y = \frac{mb^2}{(mb^2 + I)} g .
\]  

(7)

The moment of inertia of the yo-yo about the \( z \)-axis passing through the center of mass is

\[
I = \frac{1}{2} mR^2.
\]  

(8)

Now substitute Eq. (8) in to Eq. (7) yielding for the acceleration

\[
a_y = \frac{1}{(1 + R^2 / 2b^2)} g .
\]  

(9)

For a typical Yo-Yo, the acceleration is much less than that of an object in free fall.