Problem Solving Session 10

Three Dimensional Rotational Motion and Gyroscopes

A gyroscope wheel is at one end of an axle of length \( d \). The other end of the axle is suspended from a string of length \( s \). The wheel is set into motion so that it executes uniform precession in the horizontal plane. The string makes a fixed angle \( \beta \) with the vertical. The wheel has mass \( M \) and moment of inertia about its center of mass \( I_{cm} \). Its spin angular speed is \( \omega \). Neglect the mass of the shaft and the mass of the string. Assume \( \omega >> \Omega \). What is the direction and magnitude of the precession angular velocity? Express your answer in terms of in terms of \( d \), \( M \), \( \omega \), \( \beta \), \( s \), and \( g \) as needed.

Hint: Choose the center of mass as the point for calculating torque and angular momentum.
Consider a simple rigid body consisting of two particles of mass $m$ separated by a rod of length $2l$ and negligible mass. The midpoint of the rod is attached to a vertical axis that rotates with angular velocity $\vec{\omega} = \omega \hat{k}$ about the $z$-axis. The rod is skewed from the vertical at an angle $\phi$. Set time $t = 0$ when the rod is in the position shown in figure below left. At $t = \pi / \omega$ the rod has rotated to the position shown in the figure below right.

a) Find the direction and magnitude of the angular momentum about the center of mass at $t = 0$.

b) Find the direction and magnitude of the torque about the center of mass at time $t = 0$. 

W14D2-4 Stabilizing a Car

(Only for sections that did not finish this problem on W14D2).

When an automobile rounds a curve at high speed (in the figure below the car is turning left), the loading (weight distribution) on the wheels is markedly changed. For sufficiently high speeds the loading on the inside wheel goes to zero, at which point the car starts to roll over. The tendency can be avoided by mounting a large spinning flywheel on the car.

![Image of car turning left with labels r, h, cm, and v_{cm}]

car moving into page and turning left

a) What should be the sense of rotation of the flywheel to help equalize the loading? (Be sure that your method works for cars turning in either direction.)

b) Show that for a disk-shaped flywheel of mass $m_w$ and radius $R$, the requirement for equal loading is that the angular velocity of the flywheel, $\omega_z$, is related to the speed of the car $v_{cm}$ by

$$\omega_z = 2v_{cm} \frac{m}{m_w R^2},$$

where $m_c$ is the total mass of the car and flywheel, and $h$ is the height of the center of mass of the car (including the flywheel) above the road. Assume the road is unbanked.