Math Review 1: Vectors
Coordinate System

Coordinate system: used to describe the position of a point in space and consists of

1. An origin as the reference point
2. A set of coordinate axes with scales and labels
3. Choice of positive direction for each axis
4. Choice of unit vectors at each point in space

Cartesian Coordinate System
Vectors
Vector

A vector is a quantity that has both direction and magnitude. Let a vector be denoted by the symbol $\vec{A}$. The magnitude of $\vec{A}$ is denoted by $|\vec{A}| = A$. 
Application of Vectors

(1) Vectors can exist at any point $P$ in space.

(2) Vectors have direction and magnitude.

(3) Vector Equality: Any two vectors that have the same direction and magnitude are equal no matter where in space they are located.
Vector Addition

Let \( \vec{A} \) and \( \vec{B} \) be two vectors. Define a new vector \( \vec{C} = \vec{A} + \vec{B} \), the “vector addition” of \( \vec{A} \) and \( \vec{B} \) by the geometric construction shown in either figure.
Vector Decomposition

Choose a coordinate system with an origin and axes. We can decompose a vector into component vectors along each coordinate axis, for example along the x, y, and z-axes of a Cartesian coordinate system. A vector at $P$ can be decomposed into the vector sum,

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$
Unit Vectors and Components

The idea of multiplication by real numbers allows us to define a set of unit vectors at each point in space ($\hat{i}$, $\hat{j}$, $\hat{k}$) with $|\hat{i}|=1$, $|\hat{j}|=1$, $|\hat{k}|=1$

Components:

$\vec{A} = (A_x, A_y, A_z)$

$\vec{A}_x = A_x \hat{i}, \quad \vec{A}_y = A_y \hat{j}, \quad \vec{A}_z = A_z \hat{k}$

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
Vector Decomposition in Two Dimensions

Consider a vector
\[ \vec{A} = (A_x, A_y, 0) \]

x- and y components:
\[ A_x = A \cos(\theta), \quad A_y = A \sin(\theta) \]

Magnitude:
\[ A = \sqrt{A_x^2 + A_y^2} \]

Direction:
\[ \frac{A_y}{A_x} = \frac{A \sin(\theta)}{A \cos(\theta)} = \tan(\theta) \]

\[ \theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) \]
Vector Addition

\[ \vec{A} = A \cos(\theta_A) \hat{i} + A \sin(\theta_A) \hat{j} \]

\[ \vec{B} = B \cos(\theta_B) \hat{i} + B \sin(\theta_B) \hat{j} \]

Vector Sum: \[ \vec{C} = \vec{A} + \vec{B} \]

Components

\[ C_x = A_x + B_x, \quad C_y = A_y + B_y \]

\[ C_x = C \cos(\theta_C) = A \cos(\theta_A) + B \cos(\theta_B) \]

\[ C_y = C \sin(\theta_C) = A \sin(\theta_A) + B \sin(\theta_B) \]

\[ \vec{C} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} = C \cos(\theta_C) \hat{i} + C \sin(\theta_C) \hat{j} \]
Given two vectors, find:

(a) $|\vec{A}|$

(b) $|\vec{B}|$

(c) $\vec{A} + \vec{B}$

(d) $\vec{A} - \vec{B}$

\[ \vec{A} = 2\hat{i} - 3\hat{j} + 7\hat{k} \]

\[ \vec{B} = 5\hat{i} + \hat{j} + 2\hat{k} \]
From the information given in the figure above, write down a mathematical representation for the vector decomposition of the vector

\[ \mathbf{C} = \mathbf{A} - \mathbf{B} \]
An object of mass $m$ is sliding down a frictionless inclined plane that makes an angle $\theta$ with respect to the horizontal. Two forces are acting on the object, the normal force $\vec{N}$ and the gravitational force $\vec{F}_{grav} = m\vec{g}$.

Write down a mathematical expression for the vector decomposition of the sum of the two forces $\vec{N} + m\vec{g}$

i) using the coordinate system shown in figure (a),
ii) using the coordinate system shown in figure (b).
iii) Does the magnitude of the sum of the two forces depend on the choice of coordinate system?
An object of mass $m$ is sliding down a frictionless inclined plane that makes an angle $\theta$ with respect to the horizontal in terms of the angle $\theta$ and the magnitude of the acceleration $a$.

i) Draw a diagram that shows the acceleration of the object.

Write down a mathematical expression for the vector decomposition of the acceleration

ii) using the coordinate system shown in figure (a),

iii) using the coordinate system shown in figure (b).
Discussion: Newton’s Second Law

What are the advantages and disadvantages of choosing the coordinate system shown in figures (a) and (b)?
Charlie is walking in the train

Charlie is walking in the same direction as the train is moving. Two passengers, Albert and Bob, are sitting watching Charlie move.

The displacement vector observed by Albert is equal or different than the one observed by Bob?
Albert is on the platform

Charlie is walking in the same direction as the train is moving. Albert is on the platform and Bob is sitting in the train.

The displacement vector observed by Albert is equal or different than the one observed by Bob?
Preview: Vector Description of Motion

- **Position**  \( \vec{r}(t) = x(t) \hat{i} + y(t) \hat{j} \)

- **Displacement**  \( \Delta \vec{r}(t) = \Delta x(t) \hat{i} + \Delta y(t) \hat{j} \)

- **Velocity**  \( \vec{v}(t) = \frac{dx(t)}{dt} \hat{i} + \frac{dy(t)}{dt} \hat{j} \equiv v_x(t) \hat{i} + v_y(t) \hat{j} \)

- **Acceleration**  \( \vec{a}(t) = \frac{dv_x(t)}{dt} \hat{i} + \frac{dv_y(t)}{dt} \hat{j} \equiv a_x(t) \hat{i} + a_y(t) \hat{j} \)