Newton’s Second Law in Different Reference Frames
Announcements

Open up the Daily Concept Questions page on the MITx 8.01x Webpage.

Problem Set 3 due Tue Week 4 at 9 pm

No Prepset for Week 4

Exam 1 Thursday Sept 27  7:30-9:30

No class Friday Week 4
Exam Rooms

Exam 1 Thursday Sept 27  7:30-9:30

Sections L01, L02, L05
Walker Memorial Third Floor 50-340

Sections L03, L06, L07
26-100

Section L04
26-152
Conflict Exams

Conflict Exam 1 will be held Friday Morning September 28 from 8-10 am in room 24-112 and from 9-11 am in room 32-123 (note change of room)

If you have an academic conflict or a regular scheduled activity please fill out the google form at

https://docs.google.com/forms/d/e/1FAIpQLSfYHc9yU_gaXtGnsYFp5Y5HxhPtEUxnI9trg7wS5vt0cfGeTA/viewform?usp=sf_link

in which you should describe the conflict and indicate which of the times you would like to take the conflict exam.
Exam One Info

Analytic Problems: There will be three analytic problems worth 75 points on the following topics from W01D2 through W03D3:

- Kinematics in one and two dimensions with constant and non-constant acceleration for both linear and circular motion
- Newton’s Laws of Motion
- Applications of Newton’s Second Law of Motion for linear, circular motion, and continuous systems.

Concept Questions: There will be five concept questions worth twenty-five points on material from W01D2 through W04D1. Please note that there may be concept questions regarding motion as seen from different reference frames from W04D1.
Two reference frames, Frame $O$ and Frame $O'$
Origins do not coincide.

Object has different position vectors in different frames

\[ \vec{r} = \vec{R} + \vec{r}' \]
Velocity of an Object in Different Reference Frames

Suppose an object is moving; then, observers in different reference frames will measure different velocities.

Velocity of Frame $O'$ with respect to Frame $O$:
\[ \vec{V} = \frac{d\vec{R}}{dt} \]

Velocity of the object in Frame $O$:
\[ \vec{v} = \frac{d\vec{r}}{dt} \]

Velocity of the object in Frame $O'$:
\[ \vec{v}' = \frac{d\vec{r}'}{dt} \]

Velocity of the object in two different reference frames:
\[ \frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} + \frac{d\vec{r}'}{dt} \Rightarrow \vec{v} = \vec{V} + \vec{v}' \]
Group Problem: Velocity is a Reference Frame Dependent Concept

In a reference frame \( O \) fixed to the ground, two cars 1 and 2 are moving with velocities

\[
\vec{v}_1 = 30 \text{ m} \cdot \text{s}^{-1} \hat{i} \quad \text{and} \quad \vec{v}_2 = -20 \text{ m} \cdot \text{s}^{-1} \hat{i}
\]

(a) In a reference frame \( O' \) moving with car 1, what is the velocity of car 2 (make sure you give a vector expression for the velocity)?

(b) In a reference frame \( O' \) moving with car 2, what is the velocity of car 1 (make sure you give a vector expression for the velocity)?
Group Problem: Velocity of an Object in Different Reference Frames

Suppose two cars, Car 1, and Car 2, are traveling along roads that are perpendicular to each other. Reference frame $O$ is at rest with respect to the ground. Reference frame $O'$ is at rest with respect to Car 1. In reference frame $O$, Car 1 is moving in the positive $y$-direction with speed $v_1$ and Car 2 is moving in the positive $x$-direction with speed $v_2$.

a) What is the vector description of the velocity of Car 2 in reference frame $O'$?

b) What is the magnitude of the velocity of Car 2 as observed in reference frame $O'$?

c) What angle does the velocity of Car 2 make with respect to the positive $x$-direction as observed in reference frame $O'$?
Acceleration of an Object in Different Reference Frames

Suppose an object is moving; observers in different reference frames will measure different accelerations.

Acceleration of Frame $O'$ with respect to Frame $O$:
\[ \vec{A} = \frac{d\vec{V}}{dt} \]

Acceleration of the object in Frame $O$:
\[ \vec{a} = \frac{d\vec{v}}{dt} \]

Acceleration of the object in Frame $B$:
\[ \vec{a}' = \frac{d\vec{v}'}{dt} \]

Acceleration of an object in two different reference frames:
\[ \frac{d\vec{v}}{dt} = \frac{d\vec{V}}{dt} + \frac{d\vec{v}'}{dt} \Rightarrow \vec{a} = \vec{A} + \vec{a}' \]

path of moving object in reference frame $O$

\[ \vec{r} = \vec{R} + \vec{r}' \]

\[ \vec{v} = \vec{V} + \vec{v}' \]
CQ: Inertial Reference Frames

Suppose Frames $O$ and $O'$ are moving at constant speed with respect to each other.

1) An object that is moving at constant velocity in Frame $O'$ is accelerating in reference Frame $O$.

2) An object that is accelerating in Frame $O'$ is moving at constant velocity in reference Frame $O$.

3) An object that is accelerating in Frame $O'$ has the same acceleration in reference Frame $O$.

4) None of the above
Inertial Reference Frame and Newton’s Second Law

Consider an isolated object. This means that there are no physical interactions between the object and the surroundings.

According to Newton’s First Law, the isolated object will undergo uniform motion. Choose a coordinate system such that the isolated body is at rest or is moving with a constant velocity. That coordinate system is called an \textit{inertial reference frame}.

Do such coordinate systems exist? Newton’s First Law states that it is always possible to find such a coordinate system.
Inertial Reference Frame and Newton’s Second Law

Newton’s Second Law

\[ \vec{F}_{physical} = ma \]

only holds in inertial reference frames, where \( \vec{F}_{physical} \) are the forces that arise from the interactions of objects.

An isolated object in a non-inertial frame will accelerate. Therefore Newton’s Second Law does not hold in a non-inertial reference frame when you only consider the physical forces. (We will shortly see how to modify the Second Law so that the modified version holds in on-inertial reference frames.)
CQ: Inertial or Non-Inertial Reference Frames

You are in a spaceship with the engines turned off in a zero gravitational field. You are standing on a frictionless floor at rest. Now suppose you start to slide backwards. Which of the following is true immediately after you start to slide backwards.

1. The spaceship is still an inertial reference frame and has not changed its speed.

2. The spaceship is accelerating backwards.

3. The spaceship is accelerating forwards.
Non-Inertial Reference Frames

Consider an object that is accelerating in an inertial frame $O$. In that frame, Newton’s Second Law is

$$\vec{F}_{\text{physical}} = m\vec{a}$$

where $\vec{F}_{\text{physical}}$ are the physical forces acting on the object.

Now let $O'$ be an accelerated reference frame with respect to Frame $O$. The acceleration of the object in $O'$ is given by

$$\vec{a}' = \vec{a} - \vec{\Lambda}$$

Thus

$$m\vec{a}' = m\vec{a} - m\vec{\Lambda} = \vec{F}_{\text{physical}} - m\vec{\Lambda}$$
Modifying Newton’s Second Law in Non-Inertial Reference Frames: Fictitious Forces

In order to reconcile

\[ m\ddot{a}' = m\ddot{a} - m\dot{A} \]

\[ = \vec{F}_{\text{physical}} - m\dot{A} \]

with Newton’s Second Law, we can define a “fictitious force” by

\[ \vec{F}_{\text{fictitious}} = -m\dot{A} \]

Then a modified Newton’s Second Law in Frame \( O' \) becomes

\[ \vec{F}_{\text{physical}} + \vec{F}_{\text{fictitious}} = m\ddot{a}' \]

Note that the fictitious force does not arise from a real physical interaction between objects but is an artifact of analyzing motion in a non-inertial reference frame.
Summary: Newton’s Second Law in a Rotating Reference Frame

We can still apply Newton’s Second Law in a rotating reference frame but we now must consider the fictitious forces as well as the physical forces:

\[ \vec{F}_{\text{physical}} + \vec{F}_{\text{fictitious}} = m\ddot{a}' \]
CQ: Elevator

Consider a person standing in an elevator that is accelerating upward with magnitude \( a \). In the reference frame moving with the elevator, the acceleration of the person is

1. downward and equal to gravitational acceleration \( g \)
2. upward and equal to gravitational acceleration \( g \)
3. upward and equal to \( a \)
4. downward and equal to \( a \)
5. zero
Group Problem: Person in Accelerating Elevator

Consider a person standing in an elevator that is accelerating upward with magnitude \( a \).

a) In the reference frame fixed to the ground, draw a free body force diagram on the person.

b) In the reference frame moving with the elevator, draw a free body force diagram on the person including any necessary fictitious forces.

c) In both of the cases above apply Newton’s Second Law to find the normal force on the person. Do your answers agree?
Non-Inertial Rotating Reference Frames

Consider a rotating reference frame $O'$ that is rotating with respect to an inertial reference frame $O$.

Example 1: Because the earth is rotating about its axis, the earth is a non-inertial rotating reference frame. However we will often neglect the effect this rotational motion has and assume the a reference frame fixed to the ground is an inertial reference.

Example 2: A car is undergoing uniform circular motion with respect to a reference frame $O$ that is fixed to the ground. Let reference frame $O'$ be fixed to the car. Then reference frame $O'$ is a non-inertial rotating reference frame.
Fictitious Forces in Non-Inertial Rotating Reference Frames

There are fictitious forces in a reference frame rotating at a constant angular speed, for example the centrifugal force, corresponding to the fact that an object undergoing circular motion always has a non-zero centripetal acceleration.

The fictitious forces do not have as simple a form as in a linearly accelerating reference frame.

The problem is that the relationship between the accelerations of an object in an inertial frame and rotating non-inertial frame is complicated (and we will learn how to calculate this later in the course).
Car in a Turn

You are a passenger in a racecar approaching a turn after a straight-away.

As the car turns left on the circular arc at constant speed, you are pressed against the car door.

The door is exerting a force (physical force) radially inward.

The apparent force that you feel pointing radially outwards is not a physical force but is the fictitious centrifugal force and is due to the fact that you are in a non-inertial rotating reference frame.