Exam 1
Summary and Practice
Problem Slides

8.01
W04D1
Exam Rooms

Exam 1 Thursday Sept 27  7:30-9:30

Sections L01, L02, L05
Walker Memorial Third Floor 50-340

Sections L03, L06, L07
26-100

Section L04
26-152
Conflict Exams

Conflict Exam 1 will be held Friday Morning September 28 from 8-10 am in room 24-112 and from 9-11 am in room 26-152 (note change of room)

If you have an academic conflict or a regular scheduled activity please fill out the google form at

https://docs.google.com/forms/d/e/1FAIpQLSfYHc9yU_gaXtGnsYFp5Y5HxhPtEUxnI9trg7wS5vt0cfGeTA/viewform?usp=sf_link

in which you should describe the conflict and indicate which of the times you would like to take the conflict exam.
Exam One Info

Analytic Problems: There will be three analytic problems worth 75 points on the following topics from W01D2 through W03D3:

Kinematics in one and two dimensions with constant and non-constant acceleration for both linear and circular motion

Newton’s Laws of Motion

Applications of Newton’s Second Law of Motion for linear, circular motion, and continuous systems.

Concept Questions: There will be five concept questions worth twenty-five points on material from W01D2 through W04D1. Please note that there may be concept questions regarding motion as seen from different reference frames from W04D1.
Exam 1
Problem Solving Review

8.01
W03D1
Two objects of equal mass $m$ are whirling around a shaft with a constant angular velocity $\omega$. The first object is a distance $d$ from the central axis, and the second object is a distance $2d$ from the axis. You may ignore the mass of the strings and neglect the effect of gravity. What are tensions in the string between the inner object and the outer object and the string between the shaft and the inner object?
Pressure in a Compressible fluid

Consider a column of seawater of cross-sectional area $A$, with the top of the column at sea level and the bottom of the column at a depth $h = 500$ m. You may assume that column of seawater is at rest (there are no bulk motions). The atmospheric pressure at sea level is $P_0$, and the acceleration of gravity is $g$.

Choose a coordinate system such that the $z$-axis points vertically downward and $z = 0$ is at sea level. The density of the seawater varies with depth and is given by the function

$$
\rho = \rho_0(1 + bz), \quad \text{for} \quad 0 \leq z \leq h = 500 \text{ m},
$$

where $\rho_0$ is the density at $z = 0$ and $b$ is a positive constant.

a) Determine an expression for the rate of change of pressure with respect to depth, $dP/dz$. Show all your work, including all relevant free body force diagrams. Express your answer in terms of the constants $A$, $h$, $\rho_0$, $b$, $z$, and $g$ as needed.

b) Integrate the differential equation you found in part (a) to determine an expression for the pressure as a function of distance $z$ below the surface, $P(z)$. Express your answer in terms of the constants $P_0$, $A$, $h$, $\rho_0$, $b$, $z$, and $g$ as needed.
In an ultra centrifuge shown in the figure above, two liquid filled tubes are spun with a high angular speed $\omega$ about a fixed axis such that they are spinning in a horizontal plane. Each tube has cross sectional area $A$ and length $L$. (You may neglect the curvature at the end of the tube.) The open-ended side of each tube is a distance $r_0$ from the fixed axis.

Assume that the density of the fluid varies with distance from the rotational axis

$$\rho(r) = b(r - r_0)$$

where $r$ is the distance from the rotation axis with $r_0 \leq r \leq r_0 + L$, and $b$ is a positive constant.

You may ignore the effects of gravity.

A small volume $\Delta V$ of fluid of area $A$, length $\Delta r$, located a distance $r$ from the rotation axis, has mass $\Delta m = \rho(r) A \Delta r$. The pressure at the open end of the tube is atmospheric pressure, $P(r_0) = P_{\text{atm}}$.

Find an expression for the pressure in the fluid as a function of $r$. Express your answer in terms of $b, A, L, r$, $P_0$ for $P_{\text{atm}}$, and $r_0$ for $r_0$ as needed.
Block 1 and block 2, with masses $m_1$ and $m_2$, are connected by a system of massless, inextensible ropes and massless pulleys as shown above.

**Part a)** Solve for the acceleration of block 2 in terms of $m_1$, $m_2$ and $g$. Assume that "down" is positive.
RUNNER

A runner, starting from rest at $t = 0$ and $x_0 = 0$ accelerates with an $x$-component of acceleration given by

$$a_c = \begin{cases} 
A - Bt & 0 \leq t \leq t_1 = A/B \\
0 & t_1 < t \leq t_2 = 2A/B 
\end{cases}$$

where $A$ and $B$ are positive constants. Note that $A$ has SI units of $m/s^{-2}$ and $B$ has SI units of $m/s^{-3}$.

Part a) Determine an expression for the velocity, $v(t)$, of the runner as a function of time for the interval $0 \leq t \leq t_1 = A/B$.

Part b) What is the velocity of the runner at time $t_1 = A/B$?

Part c) Determine an expression for the position, $x(t)$, of the runner as a function of time for the interval $0 \leq t \leq t_1 = A/B$.

Part d) What is the position of the runner at time $t_1 = A/B$?

Part e) How far from the starting point is the runner at time, $t_2 = 2A/B$?
Coordinate System

Used to describe the position of a point in space and vectors at any point

A coordinate system consists of:

1. An origin at a particular point in space
2. A set of coordinate axes with scales and labels
3. Choice of positive direction for each axis: unit vectors
4. Choice of type: Cartesian or Polar or Spherical

Example: Cartesian One-Dimensional Coordinate System
Position and Displacement

**Position vector** points from origin to body.

\[ \vec{r}(t) = x(t)\hat{i} \]

\( x(t) \) is called the **coordinate position function**

Change in position vector of the object during the time interval

\[ \Delta t = t_2 - t_1 \]

**Displacement vector**

\[ \Delta \vec{r} \equiv [x(t_2) - x(t_1)] \hat{i} \]

\[ \equiv \Delta x(t)\hat{i} \]
Instantaneous Velocity

The $x$-component of the velocity is equal to the slope of the tangent line of the graph of $x$-component of position vs. time at time $t$. 

Mathematically, this is expressed as:

$$v_x(t) = \frac{dx}{dt}$$
Instantaneous Velocity and Differentiation

For each time interval $\Delta t$, calculate the $x$-component of the average velocity

$$v_{\text{ave},x}(t) = \frac{\Delta x}{\Delta t}$$

Take limit as $\Delta t \to 0$ generates a sequence of $x$-components of average velocity

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \equiv \frac{dx}{dt}$$

The limiting value of this sequence is $x$-component of the instantaneous velocity at time $t$.

$$v_x(t) = \frac{dx}{dt}$$
Instantaneous Acceleration

Average acceleration for time interval $\Delta t$:

$$a_{ave,x}(t) = \frac{\Delta v_x}{\Delta t}$$

Instantaneous acceleration: limit of a sequence of average accelerations

$$a_x(t) = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} \frac{v_x(t + \Delta t) - v_x(t)}{\Delta t} \equiv \frac{dv_x}{dt}$$
Velocity as a Function of Time

Suppose instead of some difference in velocity we would like to find the velocity as a function of time. Then let \( t = t_1 \)

\[
\nu_x(t) - \nu_x(t_0) = \int_{t'=t_0}^{t'=t} a_x(t') \, dt'
\]
Change in Position: Integral of Velocity

The integral of the $x$-component of the velocity vs. time is the displacement

$$x(t) - x(t_0) = \int_{t' = t_0}^{t' = t} v_x(t') \, dt'$$
Newton’s Second Law: Physics and Mathematics

\[ \vec{F} = m \vec{a} \]

physics ⇔ geometry
cause of motion ⇔ description of motion
(why) ⇔ (how)
dynamics ⇔ kinematics
Circular Motion Dynamics
Newton’s Second Law:  
Circular Motion

\[
\vec{F} = m\ddot{a}
\]

\[\begin{align*}
\text{physics} & = \text{geometry} \\
\hat{r} : & \quad F_r = -mr\omega^2 \\
& \quad F_r = -m\frac{v^2}{r} \\
\hat{\theta} : & \quad F_\theta = mr\alpha_z \\
& \quad F_\theta = mr\frac{d^2\theta}{dt^2}
\end{align*}\]
Summary: Circular Motion: Vector Description

- **Position**: \( \vec{r}(t) = r \hat{r}(t) \)

- **Angular Velocity**: \( \vec{\omega} = \omega_z \hat{k} = (d\theta / dt) \hat{k} \)

- **Angular Acceleration**: \( \vec{\alpha} = \alpha_z \hat{k} = (d\omega_z / dt) \hat{k} = (d^2\theta / dt^2) \hat{k} \)

- **Velocity**: \( \vec{v} = v_{\theta} \hat{\theta}(t) = r(d\theta / dt) \hat{\theta} \)

- **Acceleration**: \( \vec{a} = a_r \hat{r} + a_\theta \hat{\theta} \)

\[
a_r = -r\left(\frac{d\theta}{dt}\right)^2 = -\left(\frac{v^2}{r}\right), \quad a_\theta = r\left(\frac{d^2\theta}{dt^2}\right)
\]