In Class Solutions: Faraday’s Law: Changing Area

Problem: A conducting rod is pulled along two conducting rails at a constant velocity \( v \) in a uniform magnetic field \( B \).

Find:
1. Direction of induced current
2. Direction of resultant force
3. Magnitude of EMF
4. Magnitude of current
5. Power externally supplied to move at constant \( v \)

Solution:

As always, the first step is to think about the problem a little. In Faraday’s law problems, the thought should revolve along Lenz’s law. But before we even get there, how do we recognize that this is a Faraday’s law problem? There are several clues. We are asked about “induced current.” Something is moving in a field that we are told about (rather than asked to calculate). And, as you will see, this is one of the few prototypical problems for this topic.

Back to the physics. Lenz tells us that the induced current will oppose the change. Since the area of the loop is increasing, the flux into the page is increasing, and the current will act to oppose it – it will flow \( \textbf{1}) \text{ counter-clockwise} \) to make a flux out of the page.

The resultant force can also be given by Lenz’s law – it must oppose the change and hence \( \textbf{2}) \text{ be to the left} \). Alternatively you could see this using the right hand rule on an upward current in a field into the page. To find the magnitude we need to write down Faraday’s law:

\[
\mathcal{E} = -N \frac{d\Phi}{dt} = -\frac{d}{dt}(BA) = -B \frac{dA}{dt}
\]

We can jump to writing it like this because \( \textbf{1}) \) the field is perpendicular to the loop, and \( \textbf{2}) \) the \( B \) field is uniform. Now we just need an expression for the area \( A \). If the distance between the rails is \( w \) and the distance from the resistor to the rod is \( x \), then

\[
A = wx \Rightarrow \frac{dA}{dt} = w \frac{dx}{dt} = wv,
\]

so Faraday’s Law becomes

\[
\mathcal{E} = -Bwv.
\]

Recall that the minus means that the current is counterclockwise.
The current is just determined by the EMF $\mathcal{E}$ and the resistance $R$:

$$ I = \frac{|\mathcal{E}|}{R} = \frac{B w v}{R} $$

Finally, the power supplied by the force is all being dissipated in the resistor, so:

$$ P = I^2 R = \left( \frac{B w v}{R} \right)^2 R = \frac{B^2 w^2 v^2}{R}. $$