Problem 1 Spherical Dielectric Capacitor

A spherical capacitor consists of two thin conducting spherical shells and a spherical shell of dielectric material partially filling the space between the conducting shells. The inner conducting shell has radius $a$ and the outer conducting shell has radius $c$. The space between is partially filled (from $a$ out to $b$) with a material of dielectric constant $\kappa$. The spaces in the center out to radius $a$ and outside the dielectric between $b$ and $c$ are empty (vacuum).

(a) Determine the capacitance of this capacitor.

(b) When the conducting surfaces of the capacitor are connected to a battery with a potential difference $V$, determine an expression for the stored energy $U$.

(c) While the battery is still connected, the entire region between the spherical shells is filled with the material of dielectric constant $\kappa$. By what amount has the magnitude of the free charge changed on either conducting shell $\Delta Q \equiv Q_f - Q_i$?
Problem 2: Resistance of Spherical Shells Filled with Resistive Material

A resistor consists of two concentric spherical shells that is filled with a material of resistivity $\rho_r$. The inner radius is $a$ and the outer radius is $b$. What is the resistance of this resistor?
Problem 3: Short Questions

(a) Can a constant magnetic field set into motion an electron, which is initially at rest? Explain your answer.

(b) Is it possible for a constant magnetic field to alter the speed of a charged particle? What is the role of a magnetic field in a cyclotron?

(c) How can a current loop be used to determine the presence of a magnetic field in a given region of space?

(d) If a charged particle is moving in a straight line through some region of space, can you conclude that the magnetic field in that region is zero? Why or why not?

(e) List some similarities and differences between electric and magnetic forces.
Problem 4: Particle Trajectory

A particle of charge $-e$ is moving with an initial velocity $\vec{v}$ when it enters midway between two plates where there exists a uniform magnetic field pointing into the page, as shown in the figure below. You may ignore effects of the gravitational force.

(a) Is the trajectory of the particle deflected upward or downward?

(b) What is the magnitude of the velocity of the particle if it just strikes the end of the plate?
Problem 5: Particle Orbits in a Uniform Magnetic Field

The $x-y$ plane for $x < 0$ is filled with a uniform magnetic field pointing out of the page, $\mathbf{B} = 2B_0 \hat{k}$ with $B_0 > 0$, as shown. The $x-y$ plane for $x > 0$ is filled with a uniform magnetic field $\mathbf{B} = -B_0 \hat{k}$, pointing into the page, as shown. A charged particle with mass $m$ and charge $q$ is initially at the point $S$ at $x = 0$, moving in the positive $x$-direction with speed $v$. It subsequently moves counterclockwise in a circle of radius $R$, returning to $x = 0$ at point $P$, a distance $2R$ from its initial position, as shown in the sketch.

a) Is the charge positive or negative? Briefly explain your reasoning.

b) Find an expression for the radius $R$ of the trajectory shown, in terms of $v$, $m$, $q$, and $B_0$.

c) How long does the particle take to return to the plane $x = 0$ at point $P$?

d) Describe and sketch the subsequent trajectory of the particle on the figure above after it passes point $P$. Be sure to define any relevant distances in terms of $v$, $m$, $q$, and $B_0$. 
Problem 6 Mass Spectrometer

Shown below are the essentials of a commercial mass spectrometer. This device is used to measure the composition of gas samples, by measuring the abundance of species of different masses. An ion of mass $m$ and charge $q = +e$ is produced in source $S$, a chamber in which a gas discharge is taking place. The initially stationary ion leaves $S$, is accelerated by a potential difference $\Delta V > 0$, and then enters a selector chamber, $S_1$, in which there is an adjustable magnetic field $\vec{B}_1$, pointing out of the page and a deflecting electric field $\vec{E}$, pointing from positive to negative plate. Only particles of a uniform velocity $v$ leave the selector. The emerging particles at $S_2$, enter a second magnetic field $\vec{B}_2$, also pointing out of the page. The particle then moves in a semicircle, striking an electronic sensor at a distance $x$ from the entry slit. Express your answers to the questions below in terms of $E \equiv |\vec{E}|$, $e$, $x$, $m$, $B_2 \equiv |\vec{B}_2|$, and $\Delta V$.

a) What magnetic field $\vec{B}_1$ in the selector chamber is needed to insure that the particle travels straight through?

b) Find an expression for the mass of the particle after it has hit the electronic sensor at a distance $x$ from the entry slit.
Problem 7: Magnetic Field of a Ring of Current

A circular ring of radius $R$ lying in the $xy$ plane carries a steady current $I$, as shown in the figure below.

What is the magnetic field at a point $P$ on the axis of the loop, at a distance $z$ from the center?
A current loop, shown in the figure above left consists of two arc segments as shown, with a common center at $P$. One arc segment has an opening angle of 120 degrees and the other arc segment has an opening angle of 240 degrees. Two straight-line segments join the arc segments. One arc segment has radius $R$ and the other arc segment has radius $R/2$. A current $I_1$ flows clockwise in the loop in the direction as shown.

a) What is the direction and magnitude of the magnetic field $\mathbf{B}$ at the point $P$?

Two fixed conducting rails are arranged as shown in the figure above right. A metal bar of length $s$ is placed at the origin, initially held in place, and a current $I_2$ runs through the rails and bar clockwise. The bar is then released. You may assume that the length of the bar is very short and that the magnetic field you calculated in part a) is uniform over the length of the bar. You may neglect the magnetic field due to the current through the rails.

b) What is the direction and magnitude of the magnetic force acting on bar the instant it is released?
Problem 9: Torque on Circular Current Loop

A wire ring lying in the $xy$-plane with its center at the origin carries a counterclockwise current $I$. There is an external uniform magnetic field $\vec{B} = B_x \hat{i} + B_y \hat{j}$ such that $B_y > 0$ and $B_x < 0$. The magnetic moment vector $\vec{\mu}$ is perpendicular to the plane of the loop and has magnitude $\mu = IA$ and the direction is given by right-hand-rule with respect to the direction of the current. What is the direction and magnitude of the torque on the loop?
Problem 10: Gyromagnetic Ratio

A thin uniform ring of radius $R$ and mass $M$ carrying a charge $+Q$ rotates about its axis with constant angular speed $\omega$ as shown in the figure above. Find the ratio of the magnitudes of its magnetic dipole moment to its angular momentum. (This is called the gyromagnetic ratio.)
Problem 11: Ampere’s Law Co-axial Cable

A very long coaxial cable consists of a solid cylindrical inner conductor of radius \( a \), surrounded by a concentric cylindrical conducting shell of inner radius \( b \) and outer radius \( c \). The inner conductor has a non-uniform current density \( \vec{J}_{\text{inner}} = \alpha r \hat{k} \) (pointing to the left in the figure just below) where \( \alpha \) is a positive constant with units \( \text{A} \cdot \text{m}^{-3} \). The outer conductor has a uniform current density \( \vec{J}_{\text{outer}} = -\beta \hat{k} \) where \( \beta \) is a positive constant with units \( \text{A} \cdot \text{m}^{-2} \). The conductors carry equal and opposite currents of magnitude \( I_0 \).

\( \alpha \) \ and \( \beta \) in terms of \( a \), \( b \), \( c \), and \( I_0 \).

b) Determine the magnitude and direction of the magnetic field for the regions (i) \( r < a \), (ii) \( a < r < b \), (iii) \( b < r < c \), and (iv) \( r > c \). For each region, redraw the coaxial cable clearly indicating your choice of Amperian loop and associated parameters.

c) Make a graph of the magnitude of the magnetic field as a function of the distance \( r \) from the central axis of symmetry. Clearly label each axis with any relevant values.
Problem 12: Ampere’s Law Two Slabs of Current

The figure above shows two slabs of current. Both slabs of current are infinite in the $x$ and $z$ directions, and have thickness $d$ in the $y$-direction. The top slab of current is located in the region $0 < y < d$ and has a constant current density $\mathbf{J}_{\text{out}} = J \mathbf{k}$ out of the page. The bottom slab of current is located in the region $-d < y < 0$ and has a constant current density $\mathbf{J}_{\text{in}} = -J \mathbf{k}$ into the page.

a) What is the magnetic field for $|y| > d$? Justify your answer.

b) Use Ampere’s Law to find the magnetic field at $y = 0$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

c) Use Ampere’s Law to find the magnetic field for $0 < y < d$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

d) Make a plot of the $x$-component of the magnetic field as a function of the distance $y$ from the center of the two slabs.
Problem 13: Rotating Charged Cylinders

Two very long cylindrical conductors of length $a$ each open at the ends are coaxial and rotating in opposite directions along the coaxial axis with both with angular speed $\omega$. The inner cylinder is rotating in the counterclockwise direction when seen from above. The inner cylinder has radius $r_1$ and a charge $Q_1$ distributed uniformly over the surface. The outer cylinder has radius $r_2$ and a charge $Q_2$ distributed uniformly over the surface. You may ignore edge effects.

a) What are the surface current densities $\vec{K}_1$ and $\vec{K}_2$ on the two cylinders?

b) Find the magnitude and direction of the magnetic field everywhere, (i) $r < r_1$, (ii) $r_1 < r < r_2$, and (iii) $r > r_2$. 