Work, Potential Energy and Electric Potential

W03D2
Announcements

Prep Set 3 due Online Week Four Monday at 8:30 am

Sunday Tutoring 1-5 pm in 26-152

Quiz One In-class Friday February 23
Outline

Work by electrical forces

Electric potential energy

Electric potential difference

Calculating electric potential difference using Gauss’s law

Calculating electric field from electric potential function (gradient)
Work done by force for small displacement:

\[ \Delta W_j = \vec{F}_j \cdot \Delta \vec{r}_j \]

Work done by force along path from A to B:

\[ W_{AB} = \lim_{N \to \infty} \sum_{j=1}^{N} \vec{F}_j \cdot \Delta \vec{r}_j \equiv \int_{A}^{B} \vec{F} \cdot d\vec{r} \]
Conservative Forces

If the work done by a force in moving an object from point A to point B is independent of the path,

\[ W_c \equiv \int_A^B \vec{F}_c \cdot d\vec{r} \quad \text{(path independent)} \]

then the force is called a **conservative force**. The work done by the force only depends on the location of the points A and B.
Electrical Work

Electrical force on object 1 due to interaction between charged objects 1 and 2 is a conservative force due to the radially symmetric nature of Coulomb’s Law (similar to Newton’s Universal Law of Gravitation)

$$\vec{F}_{21} = k_e \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

Conservative work done by electrical force moving object 1 from A to B is **path independent**:

$$W_{AB} = \int_{A}^{B} \vec{F}_{21} \cdot d\vec{s}$$
Group Problem: Work Done by Electrical Force

A point-like charged source object (charge $q_s > 0$) is held fixed. A second point-like charged object (charge $q_1 < 0$) is initially at a distance $r_A$ from the fixed source and moves to a final distance $r_B$ from the fixed source. What is the work done by the electrical force on the moving object? Hint: What coordinate system is best suited for this problem?
Sign of Work: Negative Work

Suppose a fixed positively charged source (charge $q_s > 0$) is at the origin and a negatively charged object (charge $q_1 < 0$) moves from $A$ to $B$ with $r_A < r_B$, where $r$ is the distance from the origin, then $W < 0$.

\[
W = -k_e q_s q_1 \left( \frac{1}{r_B} - \frac{1}{r_A} \right)
\]

\[
r_A < r_B \Rightarrow \frac{1}{r_B} - \frac{1}{r_A} < 0 \text{ and } q_s q_1 < 0 \Rightarrow W < 0
\]
Group Problem: Motion of an Electron in a Uniform Electric field

Consider two oppositely uniform charged thin plates separated by a distance \( d \). The surface charge densities \( \pm \sigma \) on the plates are uniform and equal in magnitude. An electron with charge \(-e\) and mass \( m \) is released from rest at the negative plate and moves to the positive plate.

a) What is a vector expression for the electric field between the plates (neglect edge effects)?

b) What on the work done on the electron?

c) What is the speed of the electron when it reaches the positive plate?
Electric Potential Difference due to Source of Charge

Potential difference is defined to be the **negative** of the work done by the electric force per charge in moving a test object (charge $q_t$) from A to B:

$$\Delta V_{AB} \equiv -\int_A^B \left( \frac{\mathbf{F}_{st}}{q_t} \right) \cdot d\mathbf{s} = -\int_A^B \mathbf{E}_s \cdot d\mathbf{s}$$

Units: Joule/Coulomb = Volt

Here we assume that the charged test particle does not effect the source.
# How Big is a Volt?

<table>
<thead>
<tr>
<th></th>
<th>Voltage</th>
<th>Source</th>
<th>Voltage Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA Batteries</td>
<td>1.5 V</td>
<td>High Voltage</td>
<td>100 kV-700 kV</td>
</tr>
<tr>
<td>Car Batteries</td>
<td>12 V</td>
<td>Transmission Lines</td>
<td></td>
</tr>
<tr>
<td>US Outlet (AC)</td>
<td>120 V</td>
<td>Van der Graaf</td>
<td>300 kV</td>
</tr>
<tr>
<td>Distribution</td>
<td>120 V- 70 kV</td>
<td>Tesla Coil</td>
<td>500 kV</td>
</tr>
<tr>
<td>Power Lines</td>
<td></td>
<td>Lightning</td>
<td>10-1000 MV</td>
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</tbody>
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![Diagram showing the flow of energy from a generating station to various transformers and customers.](image-url)
Demonstration: Van de Graaf D29

Breakdown of dry air
33 kV/cm

Video of Tesla Coil

http://www.youtube.com/watch?v=FY-AS13fl30
Electric potential difference is therefore change in potential energy per charge in moving the test object (charge $q_t$) from A to B:

$$\Delta V_{AB} \equiv \frac{\Delta U_{AB}}{q_t} = -\int_A^B (\mathbf{F}_{st} / q_t) \cdot d\mathbf{s} = -\int_A^B \mathbf{E}_s \cdot d\mathbf{s}$$
If a positively charged particle is released from rest in an electric field, the charge will move:

1. from higher to lower electric potential resulting in an increase in potential energy.

2. from higher to lower electric potential resulting in a decrease in potential energy.

3. from lower to higher electric potential resulting in an increase in potential energy.

4. from lower to higher electric potential resulting in a decrease in potential energy.
Demonstration: Electrostatic Generator
Wimshurst Machine D14
Wimshurst Machine

A Wimshurst electrostatic generator, working on the principle of induction, generates high voltage differences and sparks between two movable electrodes. By increasing the distance between the electrodes, higher potential differences can be built up. Larger charges can be stored by connecting the knobs to Leyden jars which are component parts of the machine.

Chemical potential energy is converted to electric potential energy.
Electric Field and Potential: Effects

A charged particle, charge $q$ and mass $m$, in an electrostatic electric field will experience a force given by

$$\vec{F} = q\vec{E}$$

When a charged particle (charge $q$) is moved across an electric potential difference then the change in potential energy is

$$\Delta U = q\Delta V$$

The change in mechanical energy of the charged particle is

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = (1/2)m(v_f^2 - v_i^2) + q\Delta V$$
CQ: Motion of Negative Charged Object in External Electric Field

If a negatively charged particle is released from rest in an electric field, the charge will move

1. from higher to lower electric potential resulting in an increase in potential energy.

2. from higher to lower electric potential resulting in a decrease in potential energy.

3. from lower to higher electric potential resulting in an increase in potential energy.

4. from lower to higher electric potential resulting in a decrease in potential energy.
Using Gauss’ s Law to find Electric Potential from Electric Field

If the charge distribution has “enough” symmetry to use Gauss’ s Law to calculate the electric field, then you can calculate the electric potential difference between two points A and B

\[ V_B - V_A = - \int_{A}^{B} \vec{E} \cdot d\vec{s} \]

This is a path independent line integral.
Group Problem: Potential Difference of Two Oppositely Charged Plates

Two parallel infinite non-conducting plates lying in the xy-plane are separated by a distance d. The upper plate located at \( z = d/2 \) is uniformly positively charged with surface charge density \( +\sigma \). The lower plate located at \( z = -d/2 \) is uniformly negatively charged with surface charge density \( -\sigma \). Find the electric potential difference

\[
V\left(\frac{d}{2}\right) - V\left(-\frac{d}{2}\right)
\]
Worked Example: Spherical Shells

These two spherical shells have equal but opposite charge. Find the potential difference

\[ V(b) - V(a) = - \int_{r=a}^{r=b} \mathbf{E} \cdot d\mathbf{s} \]
Electric Potential for Nested Shells

From Gauss’s Law

\[
\vec{E}(r) = \begin{cases} \frac{kQ}{r^2} \hat{r}; & b < r < a \\ \vec{0}; & \text{elsewhere} \end{cases}
\]

Use

\[
V(b) - V(a) = - \int_0^{r=b} \vec{E} \cdot d\vec{s}
\]

where

\[
d\vec{s} = dr\hat{r}
\]

\[
V(b) - V(a) = - \int_0^{r=b} \frac{kQ}{r^2} \hat{r} \cdot dr\hat{r} = - \int_0^{r=b} \frac{kQ}{r^2} dr = \frac{kQ}{r} \bigg|_0^{r=b} - \frac{kQ}{r} \bigg|_0^{r=a} < 0
\]

\[
= \frac{kQ}{b} - \frac{kQ}{a} < 0
\]
A very long thin uniformly charged cylindrical shell (length $h$ and radius $a$) carrying a positive charge $+Q$ is surrounded by a thin uniformly charged cylindrical shell (length $h$ and radius $b$) with negative charge $-Q$, as shown in the figure. You may ignore edge effects. Find $V(b) - V(a)$. 
Deriving $E$ from $V$

\[ \Delta V = - \int_A^B \vec{E} \cdot d\vec{s} \]

Consider two points located at $A = (x,y,z)$, $B = (x+\Delta x,y,z)$

\[ \Delta \vec{s} = \Delta x \hat{i} \]

\[ \Delta V = - \int_{(x,y,z)}^{(x+\Delta x,y,z)} \vec{E} \cdot d\vec{s} \cong -\vec{E} \cdot \Delta \vec{s} = -\vec{E} \cdot (\Delta x \hat{i}) = -E_x \Delta x \]

$E_x = \text{negative of the rate of change in } V \text{ with respect to } x$

where $y$ and $z$ are held constant
Deriving $\mathbf{E}$ from $V$: gradient operator

If we do all coordinates:

$$
\mathbf{E} = - \left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)
$$

$$
= - \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) V
$$

Gradient (del) operator:

$$
\nabla \equiv \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}
$$

$$
\mathbf{E} = - \nabla V
$$
The above shows potential $V(x)$. Which is true?

1. $E_{x>0}$ is positive and $E_{x<0}$ is positive
2. $E_{x>0}$ is positive and $E_{x<0}$ is negative
3. $E_{x>0}$ is negative and $E_{x<0}$ is negative
4. $E_{x>0}$ is negative and $E_{x<0}$ is positive
A potential $V(z)$ is plotted above. It does not depend on $x$ or $y$.

a) What is the electric field for $-5 \text{ mm} < z < 5 \text{ mm}$?

b) Are there any charge distributions in the range $-5 \text{ mm} < z < 5 \text{ mm}$? If so, describe them (including sign)?