Electric Potential Difference from Electric Field

W03D2
Announcements

**Week 3:**
Problem Set 2 Due Wednesday at 11 pm
FPS week 3

**Week 4:**
LS 1 and LS 2 due Sunday at 11 pm
LS 3 and LS 4 due Tuesday at 11 pm
Problem Set 3 Due Wednesday at 11 pm

Thursday March 2 Quiz 7:30-9:00 (Topics: Week 1-Week 3)
Coulomb’s Law, Gauss’s Law, Electric Potential Difference
Outline

Work and Electric Potential Energy

Electric Potential Difference

Examples: Electric Potential Difference for Symmetric Sources of Charge

Conservation of Energy
Work, Potential Energy, Potential Difference

Conservative work on moving a test charged object (charge $q_t$) from A to B:

$$W_{AB} = \int_A^B q_t \mathbf{E} \cdot d\mathbf{r}$$

The change in potential energy of the object is:

$$\Delta U_{A,B} = U(B) - U(A) = -W_{A,B}$$

*Electric potential difference* is **defined** to be the change in potential energy per charge in moving the test object (charge $q_t$) from A to B:

$$\Delta V_{AB} = \frac{-W_{AB}}{q_t} = \frac{\Delta U_{AB}}{q_t} = -\int_A^B \mathbf{E}_s \cdot d\mathbf{s}$$

SI Units: Joule/Coulomb = Volt
# How Big is a Volt?

<table>
<thead>
<tr>
<th>Type</th>
<th>Voltage</th>
<th>Device Description</th>
<th>Voltage Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA Batteries</td>
<td>1.5 V</td>
<td>High Voltage Transmission Lines</td>
<td>100 kV-700 kV</td>
</tr>
<tr>
<td>Car Batteries</td>
<td>12 V</td>
<td>Van der Graaf</td>
<td>300 kV</td>
</tr>
<tr>
<td>US Outlet (AC)</td>
<td>120 V</td>
<td>Tesla Coil</td>
<td>500 kV</td>
</tr>
<tr>
<td>Distribution Power Lines</td>
<td>120 V-70 kV</td>
<td>Lightning</td>
<td>10-1000 MV</td>
</tr>
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</table>
CQ: Wimshurst Machine

A Wimshurst electrostatic generator, working on the principle of induction, generates high voltage differences and sparks between two movable electrodes.

Charge up the two electrodes (located at S in the figure) that are separated by a distance \( d \). They discharge when the potential difference between them is \( V \).

The electrodes are then moved further apart, separated now by a distance \( d_1 > d \) and allowed to charge up and then discharge when the potential difference is \( V_1 \).

Which of the following is true?

1. \( V_1 < V \)
2. \( V_1 = V \)
3. \( V_1 > V \)
A very thin long non-conducting plate, lying in the yz-plane, is positively charged with uniform surface charge density $\sigma > 0$.

a) Find a vector expression for the electric field for the region $x > 0$.

b) Find an expression for the electric potential difference between the points $B$ and $A$:

$$V(B) - V(A) = - \int_A^B \vec{E}_s \cdot d\vec{s}.$$ 

The point $B$ is located at $(x_B, y)$ and the point $A$ is located at $(x_A, y)$ where $x_B > x_A > 0$. 

Group Prob. Potential Difference Near a Uniformly Charged Sheet
Electric Potential Energy and Electric Potential Difference

When a particle with charge $q$ moves across a potential difference $\Delta V$ due to an external electric field, the change in the potential energy of the particle is

$$\Delta U = q\Delta V$$

Note: At each point in space the force on the charged particle is

$$\vec{F} = q\vec{E}$$
Conservation of Energy

When a charged particle (charge $q$) is moved across an electric potential difference then the change in potential energy is

$$\Delta U = q\Delta V = q(V_f - V_i)$$

Assuming no other energy changes then the change in energy of the charged particle is

$$\Delta E = \Delta K + q\Delta V$$
$$\Delta E = \frac{1}{2}m(v_f^2 - v_i^2) + q(V_f - V_i)$$
CQ: Motion of Negative Charged Object in External Electric Field

If a negatively charged particle is released from rest in an electric field, the charge will move

1. from higher to lower electric potential resulting in an increase in potential energy.

2. from higher to lower electric potential resulting in a decrease in potential energy.

3. from lower to higher electric potential resulting in an increase in potential energy.

4. from lower to higher electric potential resulting in a decrease in potential energy.
These two spherical conducting shells have equal but opposite charge.

a) Find a vector expression for the electric field as a function of the distance $r$ from the center, for the region $a \leq r \leq b$.

b) Find the potential difference between the shells

$$V(b) - V(a) = - \int_{r=a}^{r=b} \mathbf{E} \cdot d\mathbf{s}$$

c) Check your answer: do you expect it to be positive or negative?
CQ: Van de Graaf Generator

For the potential difference between two shells; set the outer shell radius \( b = \infty \), and the inner shell radius \( a \). When the sphere is charged up, the relation between the potential and the magnitude of the electric field on the surface of the sphere is

1. \( E = \frac{a}{V} \)
2. \( E = Va \)
3. \( E = \frac{V}{a} \)
4. \( E = V = 0 \)
Dem: Van de Graaf D29

For the potential difference between two shells; set $b = \infty$, $a = 11.5$ cm. Breakdown of dry air occurs when the magnitude of the electric field is $E(a) = 33$ kV/cm. Then potential at discharge is approximately

$$V(a) = \frac{kQ}{a} ; \quad \vec{E}(a) = \frac{kQ}{a^2} \hat{r} \Rightarrow V(a) = E(a)a$$

$$V(a) = (33 \text{ kV} \cdot \text{cm}^{-1})(11.5 \text{ cm}) = 3.8 \times 10^5 \text{ V}$$

$$Q = (3.3 \times 10^6 \text{ N} \cdot \text{C}^{-1})(11.5 \times 10^{-2} \text{ m})^2 / (9.0 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2})$$

$$\approx 5 \times 10^{-6} \text{ C}$$

which corresponds to about $3 \times 10^{13}$ electrons.

file:///Users/padour/Documents/Coursework/8.02%20Coursework/8.02_Spring2010_web/802TEAL3D/visualizations/electrostatics/vdgdischarge/VanDeGraffSparkFulll_smooth.mpg
A very long thin uniformly charged cylindrical shell (length h and radius a) carrying a positive charge +Q is surrounded by a thin uniformly charged cylindrical shell (length h and radius b) with negative charge -Q, as shown in the figure. You may ignore edge effects.

Find \( V(a) - V(b) \).
Appendix
Group Problem: Potential Difference of Two Oppositely Charged Plates

Two parallel very thin and long non-conducting plates, lying in the xy-plane, are separated by a distance d. The upper plate, located at \( z = d/2 \), is positively charged with uniform surface charge density \( +\sigma \). The lower plate located at \( z = -d/2 \) is negatively charged with uniform surface charge density \( -\sigma \). You may neglect any edge effects.

a) Find a vector expression for the electric field between the plates. (Don’t forget you Gaussian surface!)

b) Find an expression for the electric potential difference \( V(d/2) - V(-d/2) \)
Consider two oppositely uniform charged thin plates separated by a distance $d$. The surface charge densities $\pm \sigma$ on the plates are uniform and equal in magnitude. During the class, you calculated the potential difference between the plates.

$$V(d / 2) - V(-d / 2) = \frac{\sigma d}{\varepsilon_0}$$

An electron with charge $-e$ and mass $m$ is released from rest at the negative plate and moves to the positive plate. What is the speed of the electron when it reaches the positive plate?

1. $v = \sqrt{m\varepsilon_0 / 2e\sigma d}$    
2. $v = m\varepsilon_0 / 2e\sigma d$    
3. $v = \sqrt{2e\sigma d / m\varepsilon_0}$    
4. $v = 2e\sigma d / m\varepsilon_0$