Magnetic Fields, Magnetic Forces, and Sources of Magnetic Fields

W06D1
Announcements

Week 6:
Week 6 LS 1 and LS 2 due Sunday at 11 pm
Experiment 1 Prelab Sequences 1-2 due Tuesday at 11 pm
Problem Set 5 Due Wednesday at 11 pm
No Friday class

See MITx Exam 1 page for exam info, equation sheet, practice problems, and conflict form

Exam 1
Thursday Mar 16  7:30-9:30 pm
Conflict Exam 1 Friday Mar 17 8-10 am  or 9-11 am

Week 7:
Week 7 LS 1 and LS 2 due Sunday at 11 pm
Week 7 LS 3 and Mid-Semester Survey due Tuesday at 11 pm
Problem Set 6 Due Wed 11 pm
Exam 1 Rooms

Exam 1 Room Assignments:
50-340: Sections L01, L04, L07
32-123: Sections: L02, L05 Last Names begin A-L
26-152: Section L05 Last Names begin M-Z
34-101: Sections: L03, L08
6-120: Section: L06

Conflict Exam 1
Friday March 17 from 8:00 -10:00 am in 32-155
Friday March 17 from 9:00 -11:00 am in 32-141
Outline

Magnetic Field

Lorentz Force Law

Magnetic Force on Current Carrying Wire

Sources of Magnetic Fields

Biot-Savart Law
Magnetic Field of Sun

https://www.youtube.com/watch?v=hH9u5ql0MGw
Magnetic Field of the Earth

North magnetic pole located in southern hemisphere

http://www.youtube.com/watch?v=AtDAOxaJ4Ms
Magnetic Field Units

Force Law:  \[ \vec{F}_{q}^{B} = q\vec{v}_{q} \times \vec{B} \]

SI Units:  tesla [T]

\[
[T] = \frac{[N]}{[C][m/s]} = \frac{[N]}{[A][m]}
\]

cgs unit:  gauss [G]

1 tesla = 10^4 gauss
How Big is a Tesla?

- Earth’s Field \(5 \times 10^{-5} \text{T} = 0.5 \text{ G}\)
- Brain (at scalp) \(\approx 1 \text{ fT}\)
- Refrigerator Magnet \(\approx 1 \text{ mT}\)
- Inside MRI \(2 \text{ T}\)
- Good NMR Magnet \(18 \text{ T}\)
- Biggest in Lab \(150 \text{ T (pulsed)}\)
- LHC magnets (27 km long) \(8.4 \text{ T}\)
- Sparc Fusion HTS Magnets: \(20 \text{ T (pulsed with Energy 110 MJ)}\)
- Biggest in Pulsars \(10^8\)
Demonstration:

Magnetic Field Lines of a Bar Magnet G2
Magnetic Force on Moving Charges

Magnetic force is perpendicular to both velocity of the charge and magnetic field.

\[
\vec{F}_q^B = q\vec{v}_q \times \vec{B}
\]

Force on positive charge

Force on negative charge
CQ: Cross Product and Magnetic Force

An electron is traveling up in a magnetic field that points to the right. What is the direction of the force on the electron?

1. Up.
2. Down.
3. Left.
4. Right.
5. Into plane of figure.
6. Out of plane of figure.
A positively charged particle with charge \( +q \) and mass \( m \) is moving with speed \( v \) in a uniform magnetic field of magnitude \( B \) directed into the plane of figure will undergo circular motion. Find

1. the radius \( R \) of the orbit,
2. the period \( T \) of the motion,
3. the angular frequency \( \omega \).
4. Sketch the motion of the particle.

\[
\vec{F}_q^B = q\vec{v}_q \times \vec{B}
\]
Lorentz Force Law

Force on charged particles in electric and magnetic fields

Electric Force

\[ \vec{F}_q^E = q \vec{E} \]

Magnetic Force

\[ \vec{F}_q^B = q \vec{v}_q \times \vec{B} \]

Lorentz Force Law:

\[ \vec{F}_q = q(\vec{E} + \vec{v}_q \times \vec{B}) \]
Electrons with charge \( q = -e \) and mass \( m \) are emitted from the cathode \( C \) and accelerated toward slit \( A \) with different speeds in the direction shown. The electrons enter a region with a downward pointing electric field (magnitude \( E \)) and a magnetic field (magnitude \( B \)) that points into the plane of the figure. The electrons that travel on a straight trajectory through the plates have speed

1. \( v = B/E \)
2. \( v = (1/2)(eE/m)t^2 \)
3. \( v = 1/eB \)
4. \( v = E/B \)

\[ \mathbf{F}_q = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \]
Magnetic Force on Current-Carrying Wire

Current source infinitesimal element

\[ dq \vec{v}_{dq} = dq \frac{d \vec{s}}{dt} = dq \frac{d \vec{s}}{dt} = I \, d\vec{s} \]

Direction of \( d\vec{s} \) is the direction of \( I \).

Force on source element in external magnetic field

\[ d\vec{F}_{\text{mag}} = dq \vec{v}_{dq} \times \vec{B}_{\text{ext}} = I d\vec{s} \times \vec{B}_{\text{ext}} \]

Force on a current carrying wire in an external magnetic field

\[ \vec{F}_{\text{mag}} = \int_{\text{wire}} I d\vec{s} \times \vec{B}_{\text{ext}} \]
Demonstration:
Wire in a Magnetic Field (Jumping Wire) G8

\[ d\vec{F}_{\text{mag}} = I d\vec{s} \times \vec{B} \]
If the wire is in a uniform magnetic field then

\[ \vec{F}_{mag} = \left( \int_{\text{wire}} I \, d\vec{s} \right) \times \vec{B}_{\text{ext}} \]

where the direction of the vector \( d\vec{s} \) is the direction of the current

If the wire is also straight then

\[ \vec{F}_{mag} = I(\vec{L} \times \vec{B}_{\text{ext}}) \]

where \( \vec{L} \) is a length vector with direction equal to direction the current
Magnetic Force on Current-Carrying Wire

Current is moving charges, and we know that moving charges feel a force in a magnetic field with direction given by

$$\vec{F}_{\text{mag}} = I(\vec{L} \times \vec{B})$$

where the direction of the vector $\vec{L}$ is the direction of the current.
Moving Charges are Sources of Magnetic Fields

http://youtu.be/JmqX1GrMYnU
Magnetic Field of Moving Charge

Moving charge with velocity \( \vec{V} \) produces magnetic field at the point \( P \):

\[
\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{V} \times \hat{r}}{r^2}
\]

\( \hat{r} \): unit vector directed from charged object to \( P \)

\( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} \cdot \text{A}^{-1} \) permeability of free space

\( q > 0 \)
Demonstration:
Current is a Source of Magnetic Field
Field Generated by Wire G12
Biot-Savart Law

Current element $d\mathbf{s}$ of length $ds$ pointing in direction of current $I$ produces a magnetic field at point $P$:

$$d\mathbf{B} = \frac{\mu_0 I d\mathbf{s} \times \hat{r}}{4\pi r^2}$$

$$dq \mathbf{v}_{dq} = \frac{dq}{dt} d\mathbf{s} = I d\mathbf{s}$$

The Right-Hand Rule #2

\[
d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{s} \times \hat{r}}{r^2}
\]

\[
d\vec{B}(P) = \hat{k} \times \hat{r} = \hat{\theta}
\]

current directed out of plane of figure
A thin, straight wire carrying a current $I$ is placed along the $x$-axis, as shown in the figure above. Find a vector expression for the magnetic field at the point $P$, located along the perpendicular bisector of the wire, a distance $y$ from the wire. 

Integration Formula:

$$\int_{x'=-L/2}^{x'=L/2} \frac{dx'}{(y^2 + x'^2)^{3/2}} = \frac{1}{y^2} \left( \frac{x'}{(y^2 + x'^2)^{1/2}} \right)_{x'=-L/2}^{x'=L/2}$$
Magnetic Field of a Very Long Current Carrying Straight Wire

For the case of a field point near a very long wire with $L \gg y$, then the magnetic field is given by

$$\mathbf{B}(P) = \frac{\mu_0 I}{4\pi y} \left( \frac{L}{(y^2 + (L/2)^2)^{1/2}} \right) \mathbf{\hat{k}} \approx \frac{\mu_0 I}{2\pi y} \mathbf{\hat{k}}$$
CQ: Biot-Savart

The magnetic field at P points towards the

1. +x direction
2. +y direction
3. +z direction
4. -x direction
5. -y direction
6. -z direction
7. Field is zero
Appendix
CQ: Force between Two Parallel Wires

Consider two parallel current carrying wires with the currents running in opposite directions.

1. attracted (opposites attract?)
2. repelled (opposites repel?)
3. pushed another direction
4. not pushed – no net force
Demonstration:

Force on Series or Parallel Current Carrying Wires G9

Simulations of experiment showing parallel wires attracting
http://youtu.be/rU7QOukFjTo

or repelling:
http://youtu.be/bnbKdbXofEi
Can we understand why?

Whether they attract or repel can be seen in the shape of the created B field

http://youtu.be/5nKQjKgS9z0
http://youtu.be/nQX-BM3GCv4

Simulations of field line animations showing why showing parallel wires attract: or repel:
Summary: Comparison of Electrostatics and Magnetostatics

Electric field of stationary charge distribution

\[ \vec{E}(P) = \frac{1}{4\pi\varepsilon_0} \int_{\text{source}} dq \frac{\hat{r}_{dq,P}}{|\vec{r}_{dq,P}|^2} \]

For symmetric sources

\[ \oint \vec{E} \cdot \hat{n} \, dA = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

Force on charge

\[ \vec{F} = q\vec{E} = m\vec{a} \]

Magnetic field of quasi-static current

\[ \vec{B}(P) = \frac{\mu_0}{4\pi} \int_{\text{source}} \frac{I d\hat{s} \times \hat{r}_{dq,P}}{|\vec{r}_{dq,P}|^2} \]

For symmetric sources

\[ \oint \vec{B} \cdot d\hat{s} = \mu_0 I_{\text{enc}} \]

Force on Moving Charge

\[ \vec{F} = q\vec{v} \times \vec{B} = m\vec{a} \]
A conducting rod of uniform mass per length $\lambda$ and length $l$ is suspended by two flexible wires in a uniform magnetic field of magnitude $B$ which points out of the page and a uniform gravitational field of magnitude $g$ pointing down. If the tension in the wires is zero, what is the direction and magnitude of the current?
Vector Cross Product

Magnitude: equal to the area of the parallelogram defined by the two vectors

\[ |\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta = |\vec{A}| (|\vec{B}| \sin \theta) = (|\vec{A}| \sin \theta) |\vec{B}| \quad (0 \leq \theta \leq \pi) \]

Direction: determined by the Right-Hand-Rule
Vector Product of Unit Vectors

Unit vectors in Cartesian coordinates

\[ \hat{k} = \hat{i} \times \hat{j} \]

\[ |\hat{i} \times \hat{j}| = |\hat{i}| \| \hat{j}| \sin(\pi / 2) = 1 \]

\[ |\hat{i} \times \hat{i}| = |\hat{i}| \| \hat{i}| \sin(0) = 0 \]

\[ \hat{i} \times \hat{j} = \hat{k} \quad \hat{i} \times \hat{i} = \overrightarrow{0} \]

\[ \hat{j} \times \hat{k} = \hat{i} \quad \hat{j} \times \hat{j} = \overrightarrow{0} \]

\[ \hat{k} \times \hat{i} = \hat{j} \quad \hat{k} \times \hat{k} = \overrightarrow{0} \]