Magnetic Fields, Magnetic Forces, and Sources of Magnetic Fields
Announcements

Week 6 Prepset due Week 6 Friday 8:30 am

Sunday Tutoring 1-5 pm in 26-152

PS 5 due Week 6 Tuesday at 9 pm in boxes outside 26-152
Maxwell’s Equations: Statics

Maxwell’s Equations for Static Fields (SI units)

\[ \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \quad \oint_C \vec{E} \cdot d\vec{s} = 0 \]

\[ \oint_S \vec{B} \cdot d\vec{A} = 0 \quad \oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} \]

Lorentz force law on moving charges in electric and magnetic field

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]
Outline

Lorentz Force Law

Magnetic Force on Current Carrying Wire

Sources of Magnetic Fields

Biot-Savart Law
Lorentz Force Law

Force on charged particles in electric and magnetic fields

$$\vec{F}_{\text{elec}} = q\vec{E}$$

Electric Force

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}$$

Magnetic Force

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$
Electrons with charge \(-e\) and mass \(m\) are emitted from the cathode \(C\) and accelerated toward slit \(A\) with different speeds in the direction shown. The electrons enter a region with a downward pointing electric field (magnitude \(E\)) and a magnetic field (magnitude \(E\)) that points into the plane of the figure. The electrons that travel on a straight trajectory through the plates have speed

1. \(v = \frac{B}{E}\)
2. \(v = \frac{1}{2}\left(\frac{eE}{m}\right)t^2\)
3. \(v = \frac{1}{eB}\)
4. \(v = \frac{E}{B}\)
Magnetic Force on Current-Carrying Wire

Current source infinitesimal element

\[ dq \vec{v}_{dq} = dq \frac{d\vec{s}}{dt} = dq \frac{ds}{dt} = I d\vec{s} \]

Direction of \( d\vec{s} \) is the direction of \( I \).

Force on source element in external magnetic field

\[ d\vec{F}_{mag} = dq \vec{v}_{dq} \times \vec{B}_{ext} = I d\vec{s} \times \vec{B}_{ext} \]

Force on a current carrying wire in an external magnetic field

\[ \vec{F}_{mag} = \int_{\text{wire}} I d\vec{s} \times \vec{B}_{ext} \]
Magnetic Force on Current-Carrying Wire

If the wire is in a uniform magnetic field then

$$\vec{F}_{mag} = \left( \int_{\text{wire}} I \, d\vec{s} \right) \times \vec{B}_{ext}$$

where the direction of the vector $d\vec{s}$ is the direction of the current.

If the wire is also straight then

$$\vec{F}_{mag} = I (\vec{L} \times \vec{B}_{ext})$$

where the direction of the vector $\vec{L}$ is the direction of the current.
Magnetic Force on Current-Carrying Wire

Current is moving charges, and we know that moving charges feel a force in a magnetic field with direction given by

$$\vec{F}_{\text{mag}} = I(\vec{L} \times \vec{B})$$

where the direction of the vector $\vec{L}$ is the direction of the current.
Demonstration:
Wire in a Magnetic Field (Jumping Wire) G8

\[ d\vec{F}_{\text{mag}} = I d\vec{s} \times \vec{B} \]

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%208&show=0
Demonstration:

Series or Parallel Current Carrying Wires G9

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%209&show=0

You tube video of experiment showing parallel wires attracting
http://youtu.be/rU7QOukFjTo

or repelling:
http://youtu.be/bnbKdbXofEI
Group Problem: Current Loop

A conducting rod of uniform mass per length $\lambda$ and length $l$ is suspended by two flexible wires in a uniform magnetic field of magnitude $B$ which points out of the page and a uniform gravitational field of magnitude $g$ pointing down. If the tension in the wires is zero, what is the direction and magnitude of the current?
Sources of Magnetic Fields
What Creates Magnetic Fields: Moving Charges

http://youtu.be/JmqX1GrMYnU
Electric Field of Point Charge

An electric charge produces an electric field:

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \]

\[ \hat{r} \]: unit vector directed from the charged object to the field point \( P \)
Moving charge with velocity \( \vec{V} \) produces magnetic field at the point \( P \):

\[
\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{V} \times \hat{r}}{r^2}
\]

\( \hat{r} \): unit vector directed from charged object to \( P \)

\( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} \cdot \text{A}^{-1} \) permeability of free space
Current is a Source of Magnetic Field
Demonstration: Field Generated by Wire G12

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%2012&show=0
Continuous Moving Charge Distributions: Currents & Biot-Savart
From Charges to Currents?

\[ d\vec{B} \propto dq \vec{v}_{dq} \times \hat{r} \]
\[ = \text{[coulomb]} \frac{\text{[meter]}}{\text{[sec]}} \]
\[ = \text{[coulomb]} \frac{\text{[meter]}}{\text{[sec]}} \]

\[ dq \vec{v}_{dq} = dq \frac{d\vec{s}}{dt} = \frac{dq}{dt} d\vec{s} = I d\vec{s} \]
Biot-Savart Law

Current element $d\vec{s}$ of length $ds$ pointing in direction of current $I$ produces a magnetic field at point $P$:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

The Right-Hand Rule #2

\[ \vec{B}(P) = \hat{k} \times \hat{r} = \hat{\theta} \]

current directed out of plane of figure

\[ \text{dir } d\vec{s} = \hat{k} \]
CQ: Biot-Savart

The magnetic field at P points towards the

1. +x direction
2. +y direction
3. +z direction
4. -x direction
5. -y direction
6. -z direction
7. Field is zero
Repeat Demonstration: Parallel & Anti-Parallel Currents G9

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%209&show=0
CQ: Parallel Wires

Consider two parallel current carrying wires. With the currents running in the opposite direction, the wires are

1. attracted (opposites attract?)
2. repelled (opposites repel?)
3. pushed another direction
4. not pushed – no net force
5. I don’t know
Can we understand why?

Whether they attract or repel can be seen in the shape of the created B field

http://youtu.be/5nKQjKgS9z0

http://youtu.be/nQX-BM3GCv4

You tube videos of field line animations showing why showing parallel wires attract: or repel:
CQ: Current Carrying Coils

The above coils have
1. parallel currents that attract
2. parallel currents that repel
3. opposite currents that attract
4. opposite currents that repel
Magnetic Field Generated by a Current Loop

http://web.mit.edu/viz/EM/visualizations/magnetostatics/calculatingMagneticFields/RingMagInt/RingMagIntegration.htm
Worked Example: Ring of Radius $R$

Consider a ring with radius $R$ and current $I$. Find the direction and magnitude of magnetic field $B$ at the center (P)

1) Think about it:
   - Legs contribute nothing
     $I$ parallel to $r$
   - Ring makes field into page

2) Choose a small current element $Ids$

3) Pick your coordinates and integration variables

4) Apply Biot-Savart Law
Worked Example: Ring of Radius $R$

In the circular part of the coil...

\[
\begin{align*}
\vec{d}\vec{s} & \perp \hat{r} \rightarrow |\vec{d}\vec{s} \times \hat{r}| = ds
\end{align*}
\]

Biot-Savart:

\[
\begin{align*}
\vec{d}\vec{B} &= \frac{\mu_0 I |\vec{d}\vec{s} \times \hat{r}|}{4\pi r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{r^2} \\
&= \frac{\mu_0 I}{4\pi} \frac{R d\theta}{R^2} = \frac{\mu_0 I}{4\pi} \frac{d\theta}{R}
\end{align*}
\]
Worked Example: Ring of Radius $R$

Consider a ring with radius $R$ and current $I$

\[ dB = \frac{\mu_0 I}{4\pi} \frac{d\theta}{R} \]

\[ B = \int dB = \int_0^{2\pi} \frac{\mu_0 I}{4\pi} \frac{d\theta}{R} \]

\[ = \frac{\mu_0 I}{4\pi R} \int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R} (2\pi) \]

\[ |\vec{B}| = \frac{\mu_0 I}{2R} \quad \text{direction into page} \]
Consider a ring with radius $R$ and carrying a current $I$. What is a vector expression for the magnetic field at point $P$?