Experiment 2: Magnetic Fields, Force and Torque on a Magnetic Dipole

W07D2
Announcements

Week 7 Prepset due Friday March 24 8:30 am
Exam One will be handed back at the end of class on W07D1 Mon and Tuesday.

Students can take the exam home and compare with solutions.

**Regrading Requests:** Exams must be returned with a request to regrade in class on W07D2 Wed or Thurs. The student must clearly indicate on the cover sheet which problems they would like regraded and briefly why.

Remember regrading can both raise and lower grades.

A small sample of exams have been photocopied.

**The students must not erase answers, add to answers, or change answers in any way.**
Experiment 2: Magnetic Dipole in Helmholtz Coil

The experimental apparatus consists of two coils connected to a power supply along with a tube in which a small dipole magnet hangs from a spring that can be moved by a rod. The coils can be connected to a power supply that produces a current in the coils that creates a magnetic field.
Experiment 2 Exploration

• In this experiment we would like you to investigate:

• the magnetic field associated with a variety of coils connections;

• the torque on the magnetic dipole and direction of the force on the dipole when the dipole is placed along different points along the central axis for a two coil connections, the “Helmholtz” and “Anti-Helmholtz” configurations.
Experiment 2 Start-up Instructions

Open Week 7 sequential and select Experiment 2.

All group members enter their Kerberos ID on the login page.

Move to Section 1 and start the experiment.
Animation: Magnetic Field Generated by a Current Loop

http://web.mit.edu/viz/EM/visualizations/magnetostatics/calculatingMagneticFields/RingMagInt/RingMagIntegration.htm
Current Loops are Magnetic Dipoles

Magnetic Dipole Moment:

Magnitude: product of current and an area of loop

Direction: Perpendicular to the plane of the loop, using right hand rule with respect to direction of current

\[ \vec{\mu} = I A \hat{n} = I \vec{A} \]

http://web.mit.edu/viz/EM/visualizations/magnetostatics/calculatingMagneticFields/RingMagField/RingMagField.htm
Magnetic Field of Bar Magnet

(1) A magnet has two poles, North (N) and South (S)
(2) Magnetic field lines leave from N, end at S
Demonstration:
Magnetic Field Lines
from Bar Magnet G2

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%202&show=0
Bar Magnets Are Dipoles!

Bar magnets are:
1) source for dipole field
2) rotate to orient with external magnetic field

Is there a magnetic monopole "charge" like electric "charge"?

NO! Magnetic monopoles do not exist in isolation
Magnetic Monopoles?

Electric Dipole

\[ \vec{p} \]

When cut:

2 monopoles (charges)

Magnetic monopoles do not exist in isolation:

Our Second Maxwell’s Equation! (2 of 4)

\[ \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} \]

Gauss’ s Law

Magnetic Dipole \( \vec{\mu} \)

When cut:

2 magnetic dipoles

\[ \oint_S \vec{B} \cdot d\vec{A} = 0 \]

Magnetic Gauss’ s Law
Conservation of Magnetic Flux:

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\varepsilon_0} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]
CQ: Magnetic Field Lines

The picture shows the field lines outside a permanent magnet. The field lines inside the magnet point:

1. Up
2. Down
3. Left to right
4. Right to left
5. The field inside is zero
6. I don’t know
Review: Magnetic Force on Current-Carrying Wire

If the wire is in a uniform magnetic field then

\[ \vec{F}_{mag} = \left( \int_{\text{wire}} I \, d\vec{s} \right) \times \vec{B}_{ext} \]

where the direction of the vector \( d\vec{s} \) is the direction of the current

If the wire is also straight then

\[ \vec{F}_{mag} = I(\vec{L} \times \vec{B}_{ext}) \]

where the direction of the vector \( \vec{L} \) is the direction of the current
Torque on a Current Loop in a Uniform Magnetic Field
Worked Example: Current Loop

Place rectangular current loop in uniform $\mathbf{B}$ field. Net force is zero.

\[
d\mathbf{F}_4 = I d\mathbf{l} \times \mathbf{B} = I dy' \hat{j} \times \mathbf{B} \hat{i} = -I dy' B \hat{k}
\]

The torque on the loop is

\[
d\mathbf{\tau}_4 = \mathbf{r}_{P,4} \times d\mathbf{F}_4 = (b \hat{i} + y' \hat{j}) \times (-I dy' B \hat{k}) = b I dy' \hat{j} - I B y' dy' \hat{i}
\]

\[
\mathbf{\tau}_4 = \int_{y'=-a/2}^{y'=a/2} (b I dy' \hat{j} - I B y' dy' \hat{i}) = I a b B \hat{j}
\]
Torque on Current Loop

Magnetic moment points out of the page

\[ \vec{\mu} = Iab\hat{k} \]

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]

\[ \vec{\tau} = \mu B \hat{j} = IabB \hat{j} \]

Torque tries to align the magnetic moment vector in the direction of the magnetic field
Demonstrations

Deflection of a Compass Needle by a Magnet G1

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%201&show=0

Galvanometer principle G10

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%2010&show=0
Demonstration:
Galvanometer principle G10

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=G%2010&show=0
Starting from rest, the current ring in a uniform magnetic field will:

1. rotate clockwise, not move
2. rotate counterclockwise, not move
3. move to the right, not rotate
4. move to the left, not rotate
5. move in another direction, without rotating
6. both move and rotate
7. neither rotate nor move
Magnetic Force on Current Loop In Uniform Magnetic Field

If a current loop is placed in a uniform magnetic field then

$$\vec{F}_{\text{mag}} = \left( \oint_{\text{closed loop}} I \, d\vec{s} \right) \times \vec{B}_{\text{ext}} = \vec{0}$$

Because the vector integral of the line element around a closed path is zero

$$\oint_{\text{closed loop}} d\vec{s} = \vec{0}$$
Magnetic Force on a Dipole in a Non-Uniform Magnetic Field
Force on Dipole in Non-uniform Magnetic Field

Dipoles can feel magnetic force in a non-uniform magnetic field.

Magnetism – Bar Magnet: Like poles repel, opposite poles attract
The forces shown \( d\vec{F}_{\text{mag}} = I(d\vec{s} \times \vec{B}) \) produce a net downward force on dipole into the region of greater field strength.
Force on Magnetic Dipole

What makes the field pictured? Bar magnet below dipole, with N pole on top. It is aligned with the dipole pictured, they attract!
Force on Magnetic Dipole

\[ \vec{\mu} = I A \hat{k} \]

Special case: Along z-axis, for current loops, the magnetic field points along z-axis and the force on a dipole situated on the z-axis is

\[ F_z = \mu_z \frac{\partial B_z}{\partial z} \]

Dipole is attracted to region of greater field strength.
CQ: Dipole in Magnetic Field

The current carrying coil above will feel a net force

1. upwards
2. downwards
3. of zero
4. I don’t know
Work Done by Interaction to Rotate Magnetic Dipole

\[ \vec{\tau} = \vec{\mu} \times \vec{B} = (-\mu \cos \theta \hat{j} + \mu \sin \theta \hat{i}) \times B\hat{i} = \mu \cos \theta B\hat{k} \]

\[ W_{[0,\theta]} = \int_{\theta_0}^{\theta} \tau_z \, d\theta' = \int_{\theta_0}^{\theta} \mu B \cos \theta' \, d\theta' = \mu B \sin \theta' \bigg|_{\theta_0}^{\theta} = \mu B \sin \theta \]

\[ W_{[0,\theta]} = \vec{\mu} \cdot \vec{B} \]
Pot. Energy: Dipole in Magnetic Field

\[ \Delta U = U(\theta) - U(\theta = 0) = -W_{[0, \theta]} = -\vec{\mu} \cdot \vec{B} \]

Set zero reference point

\[ U(\theta = 0) = 0 \]

\[ U(\theta) = -\vec{\mu} \cdot \vec{B} \]

Lowest energy state (aligned): 

\[ U(\theta = \pi / 2) = -\mu B \]

Highest energy state (anti-aligned):

\[ U(\theta = -\pi / 2) = \mu B \]
Force on Magnetic Dipole

\[ \vec{\mu} = I \vec{A} \]

\[ U_{Dipole} = -\vec{\mu} \cdot \vec{B} ; \quad U(\theta = 0) = 0 \]

\[ \vec{F} = -\nabla(-\vec{\mu} \cdot \vec{B}) \Rightarrow \vec{F} = \nabla(\vec{\mu} \cdot \vec{B}) \]

Special case: Magnetic field points along z-axis

\[ F_z = \mu_z \frac{\partial B_z}{\partial z} \]

Dipole is attracted to region of greater field strength
Experiment 2: Concept Questions
CQ.: Magnetic Dipole in Helmholtz Coil

A dipole is initially pointing along the positive x-direction and located above the two coils ($z > 0$). A short time later, the dipole will feel:

1. a force but not a torque.
2. a torque but not a force.
3. both a torque and a force.
4. neither force nor torque.
CQ: Dipole in Anti-Helmholtz Coil

A dipole is initially pointing along the positive z-direction and located at the center of an anti-Helmholtz coil (z = 0), with the z-component of the magnetic field shown in the figure on the lower right. A short time later the dipole will feel

1. a force but not a torque.
2. a torque but not a force.
3. both a torque and a force.
4. neither force nor torque.