Faraday’s Law

W09D1
Outline

Magnetic Flux
Electromotive Force
Faraday’s Law
Induced Electric Fields
Applications of Faraday’s Law
Problem Solving
Lenz’s Law
Announcements

Welcome back.

PS 7 due Week Nine Wednesday at 9 pm in boxes outside 26-152

Prepset Week 9 due online Week 9 Friday 8:30 am

Sunday Tutoring in 26-152 from 1-5 pm
Demonstration: Jumping Ring

An aluminum ring jumps into the air when the solenoid beneath it is energized

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H%2022&show=0
What is Going On?

This is a dramatic example of Faraday’s Law and Lenz’s Law: When current is turned on through the solenoid the created magnetic field tries to permeate the conducting aluminum ring, currents are induced in the ring to try to keep this from happening, and the ring is repelled upwards.
Demo: Electromagnetic Induction
Move a loop

Moving current loop through magnetic field

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H%203&show=0
Faraday’s Law Applet: Move Loop Keep Magnet Fixed

http://public.mitx.mit.edu/gwt-teal/FaradaysLaw2.html
Magnetic Flux Thru Wire Loop

(1) Uniform $\vec{B}$

$$\Phi_B = B_\perp A = B A \cos \theta$$

$$\Phi_B = \vec{B} \cdot \vec{A}$$

Product of perpendicular component of magnetic field and area

(2) Non-Uniform $\vec{B}$

$$\Phi_B = \iint_S \vec{B} \cdot d\vec{A}$$
Electromotive Force (emf) in a current loop

When a wire loop is moved with a velocity \( \vec{v} \) through a magnetic field. There is a magnetic force per charge equal to

\[
\frac{\vec{F}_{\text{mag}}}{q} = \vec{v} \times \vec{B}
\]

The line integral of the magnetic force around the loop at a fixed instant in time is defined to be the electromotive force (emf).

\[
\mathcal{E} = \frac{1}{q} \oint_{\text{closed path}} \vec{F}_{\text{mag}} \cdot d\vec{s}
\]
Electromotive Force (emf) in a Current loop

Emf generalizes to any force per charge integrated around a closed path.

\[ \mathcal{E} = \frac{1}{q_{\text{closed path}}} \oint \vec{F} \cdot d\vec{s} \]

Emf has the same dimensions as electric potential rather than force, so the SI unit is the volt.

If the closed path is a circuit with resistance R then the electromotive force will cause a current to flow according to

\[ \mathcal{E} = IR \]
Motional emf:

Pull a loop with speed $v$ through uniform magnetic field (shaded area). Magnetic force on moving charge is

$$\vec{F}_{mag} / q = (v\hat{i} + u\hat{j}) \times B(-\hat{k}) = vB\hat{j} - uB\hat{i}$$

At one instant in time, electromotive force is (integrating clockwise)

$$\mathcal{E} = \frac{1}{q} \oint \vec{F}_{mag} \cdot d\vec{s} = \int_{y=0}^{y=w} (vB\hat{j} - uB\hat{i}) \cdot dy \hat{j} = vBw$$
Changing Magnetic Flux: Motional emf

When the loop moves a distance $\Delta s = v\Delta t$, magnetic flux through loop is decreasing by

$$\Delta \Phi^B = -Bw\Delta s$$

$$-\frac{\Delta \Phi^B}{\Delta t} = \frac{Bw\Delta s}{\Delta t} = \frac{Bwv\Delta t}{\Delta t} = vBw$$
Electromotive Force and Changing Magnetic Flux

Electromotive force is equal to the negative rate of change of the magnetic flux

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
CQ: Moving Loop in Uniform Field I

In the figure to the right, the shaded region represents non-zero magnetic field directed out of the plane of the figure. A rectangular wire loop is pulled upward with speed \( v \). At the instant shown in the figure, there is

1. no current in the loop.
2. an induced current in the loop.
Group Problem: Changing Magnetic Flux

Conducting rod pulled along two conducting rails in a uniform magnetic field of magnitude \( B \) at constant speed \( v \). Assume the resistance has value \( R \).

a) Calculate the magnitude of the derivative of the magnetic flux with respect to time.

b) What is the magnitude of the induced current?
Electromotive Force (emf) and work

Because the electromotive force is defined as an integral at one instant in time it is not magnetic work per charge. **Recall that magnetic fields do no work!**

The work done per charge by the pulling force on the loop is equal to the electromotive force.

\[
\frac{1}{q} \int_{i}^{f} \vec{F}^p \cdot d\vec{s} = vBw = \mathcal{E}
\]
In the figure to the right, the shaded region represents non-zero magnetic field directed out of the plane of the figure. A rectangular wire loop is pulled to the left with speed $v$. At the instant shown in the figure, there is

1. no current in the loop.
2. a current in the loop.
Emf is equal to the gravitational work per charge on the falling ring
Magnet Moving Through a Fixed Loop
Magnet Falling Through Ring
Demonstration: Magnet falling through copper pipe

Demonstration: Magnet Falling Through Plastic Tube, Aluminum Tube, and Copper Tube

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H%2016&show=0
Demo: Electromagnetic Induction
Move magnet with current loop fixed

Moving magnet through fixed current loop

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H%203&show=0
Faraday’s Law Applet: Move Magnet Keep Loop Fixed

http://public.mitx.mit.edu/gwt-teal/FaradaysLaw2.html
What is the emf?

Conducting loop, with resistance $R$, is at rest, and magnet is moving as shown in figure, resulting in a changing magnetic flux through the loop and an induced current in the loop.

\[ \mathcal{E} = I_{\text{ind}} R \]
An induced electric field appears in the current loop resulting in an electric force on the charges in the conductor. Hence there is an electromotive force that is equal to the change in magnetic flux

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \int \mathbf{B} \cdot d\mathbf{A}$$

The induced current is then equal to

$$\mathcal{E} = IR$$
Faraday’s Law of Induction

If $C$ is a stationary closed curve and $S$ is a surface spanning $C$ then

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{A}$$

The changing magnetic flux through $S$ induces a non-electrostatic electric field whose line integral around $C$ is non-zero

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
Electric Guitar: Time Changing Magnetic Field Induces an Electric Field

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H%2032&show=0
Electric Guitar: Time Changing
Magnetic Field Induces an Electric Field
Sign Conventions: Right Hand Rule

\[ \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A} \]

By the right hand rule (RHR) clockwise integration direction for line integral of electric field (emf) requires that unit normal points into plane of figure for magnetic flux surface integral

Magnetic flux positive into page, negative out of page
Sign Conventions: Right Hand Rule

By the right hand rule (RHR) counterclockwise integration direction for line integral of electric field (emf) requires that unit normal points out of plane of figure for magnetic flux surface integral.

Magnetic flux positive out of page, negative into page.
CQ: Solenoid

At time $t$: the figure on the right shows a side view of a section of a very long solenoid with radius $R$ carrying current $I$ with magnetic field pointing up. The figure below right shows a top view of the electric field inside the solenoid at a radius $r$ and the direction of the magnetic field. In the solenoid, the current is

1. increasing in time.
2. constant.
3. decreasing in time.
4. cannot tell without more information.
Group Problem: Induced Electric Field

Consider a uniform magnetic field which points into the page and is confined to a circular region with radius $R$. Suppose the magnitude increases with time, i.e. $\frac{dB}{dt} > 0$. Find the magnitude and direction of the electric field in the regions (i) $r < R$, and (ii) $r > R$. (iii) Plot the magnitude of the electric field as a function $r$. 

\[ \oint_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \]
Lenz’s Law

Induced EMF is in direction that \textit{opposes the change} in flux that caused it. Induced current, torque, or force is always directed to oppose the change that caused it.

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]
For a Closed Conducting Path

Induced current is the source of induced magnetic flux that opposes the change in external magnetic flux.

\[ I_{\text{ind}} R = -\frac{d\Phi_B}{dt} \]
Direction of Induced Current

For a **counterclockwise integration direction** for the line integral hence a **upwards unit normal** for the surface integral

\[ \varepsilon = -\frac{d\Phi_B}{dt} > 0 \]

⇒ \( I_{\text{ind}} \) **counterclockwise**

\[ \varepsilon = -\frac{d\Phi_B}{dt} < 0 \]

⇒ \( I_{\text{ind}} \) **is clockwise**
Ways to Induce Changing Magnetic Flux

\[ \mathcal{E} = -\frac{d}{dt} \int \int_{\text{open surface}} \mathbf{B} \cdot \hat{n} \, da \]

Quantities which can vary with time:

- Area A enclosed by the loop with non-zero B
- Magnitude of B
- Loop moving through non-uniform B
- Angle between B and normal vector to loop
CQ: Moving Loop in Uniform Field

Lenz’s Law

In the figure to the right, the shaded region represents non-zero magnetic field directed out of the plane of the figure. A rectangular wire loop is pulled upward with speed \( v \). At the instant shown in the figure, there is

1. a counterclockwise current in the loop.
2. a clockwise current in the loop.
3. There is no current in the loop.
CQ: Magnetic Field Changing in Time

Lenz’s Law

The magnetic field through a wire loop is pointed upwards and *increasing* with time. The induced current in the coil is

1. clockwise as seen from the top.
2. counterclockwise as seen from the top.
3. zero.

\[
\frac{d\vec{B}}{dt} > 0
\]

\(\Phi\) is up and increasing
CQ: Lenz’s Law Moving Loop

A circuit in the form of a rectangular piece of wire is pulled away from a long wire carrying current $I$ in the direction shown in the sketch. The induced current in the rectangular circuit is

1. Clockwise
2. Counterclockwise
3. Neither, the current is zero