DC Circuits

W10D1
Exam 2 Announcements

**Exam Two:** Thursday 19 April 7:30-9:30 pm

Conflict Exam 2 will be held Friday April 20 from 8-10 am in 36-156 and from 9-11 am in 34-101

If you have an academic conflict or a regular scheduled activity like MITSO, please fill out the online google form at

https://docs.google.com/forms/d/e/1FAIpQLSeIq2YVhezKUCYIkXQoz8CsnBJ6bMYo6UCNFspXXASmUKL8aw/viewform?usp=sf_link
Outline

DC Circuits
Kirchoff’s Laws
Electrical Power
Measuring Voltage and Current
Examples of DC Circuits
## Symbols for Circuit Elements

<table>
<thead>
<tr>
<th>Circuit Element</th>
<th>Symbol</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery</td>
<td>$\varepsilon$</td>
<td>![Battery Symbol](image]</td>
</tr>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>![Resistor](image]</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$C$</td>
<td>![Capacitor](image]</td>
</tr>
<tr>
<td>Inductor</td>
<td>$L$</td>
<td>![Inductor](image]</td>
</tr>
<tr>
<td>Switch</td>
<td>$S$</td>
<td>![Switch](image]</td>
</tr>
<tr>
<td>Equipotential</td>
<td></td>
<td>![Equipotential](image]</td>
</tr>
<tr>
<td>Junction</td>
<td></td>
<td>![Junction](image]</td>
</tr>
<tr>
<td>Branch</td>
<td></td>
<td>![Branch](image]</td>
</tr>
</tbody>
</table>
Electromotive Force (EMF)

The work per unit charge around a closed path done is called electromotive force EMF. This is a bad name because it is not a force. Let $\vec{f}_s$ denote the force per unit charge, then the EMF is

$$\mathcal{E} = \oint_{\text{closed path}} \vec{f}_s \cdot d\vec{s}$$

If a conducting closed path is present then

$$\mathcal{E} = IR$$
Ideal Battery: emf

1) Inside Battery: non-zero chemical force $\vec{f}_s$ inside battery (zero outside), moves charges through a region in which static electric field $\vec{E}_{static}$ opposes motion $\vec{E}_{static} = -\vec{f}_s$

2) Ideal battery: net force on charges is zero

3) Potential difference between its terminals

$$V(+) - V(-) = -\int_{-}^{+} \vec{E}_{static} \cdot d\vec{s} = \int_{-}^{+} \vec{f}_s \cdot d\vec{s}$$

4) Extend path through external circuit where $\vec{f}_s = \vec{0}$

$$V(+) - V(-) = \int_{-}^{+} \vec{f}_s \cdot d\vec{s} = \oint \vec{f}_s \cdot d\vec{s} = \varepsilon$$
Sum of Potential Differences Around a Closed Path

Sum of potential differences across all elements around any closed circuit loop must be zero.

\[ \sum \Delta V_i = -\oint_{\text{Closed Path}} \bar{E}_{\text{static}} \cdot d\bar{s} = 0 \]
Electrostatics vs. Electrodynamics

In electrostatics: “Kirchoff’s Law”

\[
\oint_{\text{Closed Path}} \mathbf{E}_{\text{static}} \cdot d\mathbf{s} = 0
\]

\[
\sum_i \Delta V_i = 0
\]

When magnetic fields are changing in times: “Faraday’s Law”

\[
\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}
\]
Sign Conventions - Battery

Circulating from the negative to positive terminal of a battery increases your potential

\[ \Delta V = +\varepsilon \]

Circulating from the positive to negative terminal of a battery decreases your potential

\[ \Delta V = -\varepsilon \]
Demonstration: Five Different Types of Resistance F13

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=F_13&show=0
Sign Conventions - Resistor

Direction of current is the same as direction of electric field which points to lower electric potential.

Circulating across a resistor in the direction of current decreases the potential.

\[ \Delta V = -IR \]

Circulating across a resistor in the opposite direction of current increases your potential.

\[ \Delta V = +IR \]
Internal Resistance

Real batteries have an internal resistance, \( r \), which is small but non-zero.

\[ \Delta V = V_b - V_a = \varepsilon - I r \]

(Even if you short the leads you don’t get infinite current)
Current Conservation

Sum of currents entering any junction in a circuit must equal sum of currents leaving that junction.

\[ I_1 = I_2 + I_3 \]
Series vs. Parallel

Series

Parallel
Resistors In Series

The same current $I$ must flow through both resistors

\[ \Delta V = I (R_1 + R_2) = I R_{eq} \]

\[ R_{eq} = R_1 + R_2 \]
CQ: Bulbs & Batteries

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected

1. is higher.
2. is lower.
3. is the same.
Resistors In Parallel

Voltage drop across the resistors must be the same

\[ \Delta V = \Delta V_1 = \Delta V_2 = I_1 R_1 = I_2 R_2 = IR_{eq} \]

\[ I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \frac{\Delta V}{R_{eq}} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]
CQ: Bulbs & Batteries

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected:

1. is higher.
2. is lower.
3. is the same.
Group Problem: Four Resistors

Four resistors are connected to a battery as shown in the figure. The current in the battery is $I$, the battery emf is $\varepsilon$, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, $R_4 = 3R$.

Determine the current in each resistor in terms of $I$. 
CQ: Resistors In Parallel

Suppose that \( R_1 \gg R_2 \). Then the equivalent resistance

1. \( R_{eq} \approx R_2 \)

2. \( R_{eq} \approx R_1 \)

3. \( R_{eq} \approx R_2 / 2 \)

4. \( R_{eq} \approx R_1 / 2 \)
Reference System for Circuit Analysis

1. Simplify resistors in series/parallel.
2. Assign current and current direction in each branch.
3. Assign circulation direction for each loop.
4. Circulation direction across each element defines “before” and “after” side of element.
5. Define electric potential difference across a circuit element by

\[ \Delta V \equiv V(\text{after}) - V(\text{before}) \equiv V(a) - V(b) \]
Worked Example: Circuit

What is current through each branch of the circuit shown in the figure below?
Worked Example: Simple Circuit

Current conservation at a:

\[ I_1 = I_2 + I_3 \]

Lower Loop: Start at \( a \) and circulate clockwise. Then

\[ -I_2 2R_1 + I_3 3R_1 + \varepsilon = 0 \Rightarrow I_3 = \left( \frac{2}{3} \right) I_2 - \left( \frac{\varepsilon}{3R_1} \right) \]

Upper Loop: Start at \( a \) and circulate counterclockwise. Then

\[ -I_2 2R_1 - I_1 R_1 + 2\varepsilon = 0 \Rightarrow I_1 = \left( \frac{2\varepsilon}{R_1} \right) - 2I_2 \]

\[ I_1 = I_2 + I_3 \Rightarrow \left( \frac{2\varepsilon}{R_1} \right) - I_2 = I_2 + \left( \frac{2}{3} \right) I_2 - \left( \frac{\varepsilon}{3R_1} \right) \]

Solve for \( I_2 \):

\[ I_2 = \frac{7\varepsilon}{11R_1} \Rightarrow I_3 = \frac{\varepsilon}{11R_1} \text{ and } I_1 = \frac{8\varepsilon}{11R_1} \]
Group Problem: Multiloop Circuit

Assume the emf of the batteries and resistances are all given. Using the circuit laws, determine a set of equations that can be solved for the currents in each branch.
Measuring Voltage & Current
Measuring Potential Difference

A voltmeter must be hooked in \textit{parallel} across the element you want to measure the potential difference across.

\[
\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R_V} \approx \frac{1}{R}
\]

Voltmeters have a very large resistance, so that they don’t affect the circuit too much.
Measuring Current

An ammeter must be hooked in series with the element you want to measure the current through.

\[ R_{eq} = R + R_A \approx R \]

Ammeters have a very low resistance, so that they don’t affect the circuit too much.
CQ: Measuring Current

If $R_1 > R_2$, compare the currents measured by the three ammeters:

1. $I_1 > I_2 > I_3$
2. $I_2 > I_1 > I_3$
3. $I_3 > I_1 > I_2$
4. $I_3 > I_2 > I_1$
5. $I_3 > I_1 = I_2$
6. None of the above
7. Not enough information is given.
Measuring Resistance

An ohmmeter must be hooked in *parallel* across the element you want to measure the resistance of.

\[ \Omega \]

Here we are measuring \( R_1 \)

Ohmmeters apply a voltage and measure the current that flows. They typically won’t work if the resistor is powered (connected to a battery).
Group Problem: Multiloop Circuit

Assume the $V_{in}$ of the batteries and resistances $R_1$ and $R_2$ are all given. Determine an expression for the ratio $V_{out} / V_{in}$.
Hands-on Measurements

Each table pick up a multimeter, a resistor, a light bulb, a battery, and a single wire with stripped ends from the table next to the lectern. Do the following three activities:

1. Measure the resistance of the resistor
2. Measure the voltage of the battery
3. Make the light bulb light.
Electrical Power

Power is change in energy per unit time

So power to move current through circuit elements:

\[
P = \frac{d}{dt} (\Delta U) = \frac{d}{dt} (q \Delta V) = \frac{dq}{dt} \Delta V
\]

\[
P = I \Delta V
\]
Power - Battery

Moving from the negative to positive terminal of a battery increases your potential. If current flows in that direction the battery supplies power

\[ P_{\text{supplied}} = I \Delta V = I \varepsilon \]
Power – Resistor (Joule Heating)

Moving across a resistor in the direction of current decreases your potential. Resistors always dissipate power

\[ P_{\text{dissipated}} = I |\Delta V| = I^2 R = \frac{\Delta V^2}{R} \]

\[ \Delta V = -IR \]
CQ: Power

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the power output from the battery (compared to when only one bulb was connected)

1. Is four times higher
2. Is twice as high
3. Is the same
4. Is half as much
5. Is $\frac{1}{4}$ as much
CQ: Power

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the light (power) from the first bulb (compared to when only one bulb was connected)

1. Is four times higher
2. Is twice as high
3. Is the same
4. Is half as much
5. Is ¼ as much