1) Answers will vary.

2) A) Voters will allow the politician to steal $\bar{x}$ per period. If he steals more than that he will be kicked out, and he would never want to steal less. If a politician steals, he will take everything, so to make him indifferent between stealing everything and staying in office,

$$1 = \frac{\bar{x}}{(1-\beta)}$$

Which implies,

$$\bar{x} = (1-\beta)$$

Voters will replace a politician if he ever takes more than $(1-\beta)$. The value of staying in office for the politician is exactly equal to what he would get if he stole this period, 1. The politician chooses to take exactly $(1-\beta)$ each period, and is never replaced.

B) The voters will replace the politician iff $c_t < \bar{c}$. This level $\bar{c} > \theta$ will mean the voters will kick the politician out of office sometimes even if the politician did not steal anything, because it raises the citizen’s utility in all other states of nature. The politician will have a reservation level $\theta^*$, above which he will take $1 - \frac{\bar{c}}{\theta}$, and below $\theta^*$ the politician will take everything.

The politician then faces

$$v_t = \int_{\theta}^{\theta^*} \frac{1}{\theta - \theta} d(\theta) + \int_{\theta^*}^{\bar{c}} (1 - \frac{\bar{c}}{\theta} + \beta v_{t+1} \theta - \theta \theta^* - \beta v_{t+1} \theta^* - \bar{c} \ln \frac{\theta}{\theta^*})$$

$$v_t = \frac{1}{\theta - \theta} [\theta^* - \theta] + \frac{1}{\theta - \theta} [\bar{c} + \beta v_{t+1} \theta - \theta^* - \beta v_{t+1} \theta^*] - \beta \ln \frac{\theta}{\theta^*}$$

$$v_t = \frac{1}{\theta - \theta} [\theta^* - \theta + \bar{c} \theta - \theta^* + \beta v_{t+1} (\theta - \theta^*) - \bar{c} \ln \frac{\theta}{\theta^*}]$$

$$v_t = 1 + \beta v_{t+1} \frac{\theta - \theta^*}{\theta - \theta} - \bar{c} \frac{\ln \frac{\theta}{\theta^*}}{\theta - \theta}$$

let $v_{t+1} = v_t$

$$v_t = \frac{1 - \beta \frac{\theta - \theta^*}{\theta - \theta}}{1 - \bar{c} \frac{\theta - \theta^*}{\theta - \theta}}$$
\[ v_t = \frac{\bar{\theta} - \theta - c \ln \frac{\bar{\theta}}{\theta^*}}{\bar{\theta} - \theta - \beta(\bar{\theta} - \theta^*)} \]

We know at \( \theta^* \) the politician is indifferent between stealing it all and taking \( 1 - \frac{c}{\bar{\theta}} \) and staying in office, so

\[
1 = 1 - \frac{c}{\theta^*} + \beta \frac{\ln \frac{\bar{\theta}}{\theta^*}}{1 - \beta \frac{\bar{\theta} - \theta^*}{\bar{\theta} - \theta}}
\]

\[
\frac{c}{\theta^*} (1 - \beta \frac{\bar{\theta} - \theta^*}{\bar{\theta} - \theta}) = \beta (1 - \frac{\ln \frac{\bar{\theta}}{\theta^*}}{\bar{\theta} - \theta})
\]

\[
\frac{1}{\theta^*} \frac{\beta - \theta^*}{\theta - \theta} + \beta \frac{\ln \frac{\bar{\theta}}{\theta^*}}{\bar{\theta} - \theta} = \beta
\]

\[
\frac{\beta \theta^* (\bar{\theta} - \theta)}{(1 - \beta)\bar{\theta} - \theta + \beta \theta^* + \beta \theta^* \ln \frac{\bar{\theta}}{\theta^*}}
\]

Next, use Stirling’s Approximation, \( \ln(n+\varepsilon) = \ln(n) + \frac{\varepsilon}{n} + \frac{1}{2n^2} + \ldots \)

\[
\ln(\frac{\bar{\theta}}{\theta^*}) = \ln(1 + \frac{\bar{\theta} - \theta^*}{\theta^*}) \approx \frac{\bar{\theta} - \theta^*}{\theta^*}
\]

\[
\frac{\beta \theta^* (\bar{\theta} - \theta)}{(1 - \beta)\bar{\theta} - \theta + \beta \theta^* + \beta \theta^* \frac{\bar{\theta} - \theta^*}{\theta^*}}
\]

\[
\frac{\beta \theta^* (\bar{\theta} - \theta)}{\bar{\theta} - \theta} = \frac{\beta \theta^*}{\bar{\theta} - \theta^*}
\]

Voters will maximize their expected payout, \( c \frac{\bar{\theta} - \theta^*}{\theta - \theta} \),
\[
\max_{\theta^*} \left( \beta \theta^* \right) \frac{\bar{\theta} - \theta^*}{\bar{\theta} - \theta} \\
\beta \bar{\theta} - 2\beta \theta^* = 0 \\
\theta^* = \frac{\bar{\theta}}{2}
\]

Which means \( c = \frac{\beta \bar{\theta}}{2} \)

The voters will replace the politician if \( c < \frac{\beta \bar{\theta}}{2} \). The value of remaining in office for the politician is \( v_t = \frac{\bar{\theta} - \theta - c \ln \frac{\bar{\theta}}{\theta^*}}{\bar{\theta} - \theta - \beta (\theta - \theta^*)} \) and the politician's strategy is to take \( 1 - \frac{c}{\theta} \) if \( \theta > \theta^* \), and 1 otherwise.

C) Now both the politician and the public know \( \theta \). This will allow the voters to have their decision to replace rule depend on \( \theta \). For any known value of \( \theta \), voters can give the politician just enough so that he would rather take the fraction offered and the return to staying in office, than steal everything today. Hence \( c \) will be a function of \( \theta \), as too will \( 1 - \frac{c(\theta)}{\theta} \), which is what the politician gets to keep. Further, it seems intuitive that it is cheaper on the citizens to give more to the politician when \( \theta \) is low than when \( \theta \) is high. At an extreme, this means setting \( x(\theta) = 1 \) [\( c(\theta) = 0 \)] for \( \theta < \theta^* \) and \( x(\theta) = 0 \) [\( c(\theta) = 0 \)] for \( \theta > \theta^* \).

In every period the politician has the option of taking what he is given and having the present value of his expected future returns of staying in office, or taking everything. Define the present value of the discounted expected value of staying in office \( \equiv v \). Then,

At \( \theta^* \) the value to the politician of stealing everything is 1

And the value of staying in office is \( x(\theta^*) + \beta v = \beta v \)

Which implies,

\[
1 \leq \beta v \text{ for } \theta^*
\]

\[
v_t = \int_{\theta}^{\theta^*} \frac{1}{\theta - \bar{\theta}} d(\theta) + \int_{\theta^*}^{\infty} 0 \frac{1}{\theta - \bar{\theta}} d(\theta) + \beta v_{t+1}
\]

\[
v_t = \frac{\theta^* - \theta}{\theta - \bar{\theta}} + \beta v_{t+1}
\]

let \( v_{t+1} = v_t \), and we know \( 1 = \beta v \)

\[
(1 - \beta)v_t = \frac{\theta^* - \theta}{\theta - \bar{\theta}}
\]
\[
\frac{1}{\beta} \cdot (1)(\bar{\theta} - \bar{\theta}) + \bar{\theta} = \theta^*
\]
\[
\theta^* = \frac{(1 - \beta)(\bar{\theta} - \bar{\theta})}{\beta} + \bar{\theta}
\]

The voters optimal decision to replace rule is thus: replace the politician if \( \theta \geq \theta^* \) and \( c_t \neq \theta \). The politician's value of remaining in office is \( 1/\beta \). The politician's optimal strategy is consume 1 if \( \theta < \theta^* \), and consume 0 if \( \theta \geq \theta^* \).

Notice that the value to the citizens from this threshold rule is greater than if they contracted with the politician directly on \( x \). This makes since, as under the threshold rule, the voters are giving utility to the politician only in low realizations of \( \theta \), and receive all of \( \theta \) in high realization periods. If citizens contracted directly on \( x \), then the cutoff threshold would be that of part A), namely \( \bar{x} = (1 - \beta) \). When contracting directly on \( x \), voters pay off the politician an equal proportion, regardless of the state of \( \theta \) (i.e. without regard to cost).

Expected per period voter utility under threshold rule =
\[
\int^\beta \frac{\theta}{\theta - \bar{\theta}} d(\theta) = \frac{1}{\bar{\theta} - \bar{\theta}} \left[ \frac{1}{2} \bar{\theta}^2 - \frac{1}{2} (\theta^*)^2 \right]
\]
\[
= \frac{\bar{\theta}^2 - \theta^2}{2(\theta - \bar{\theta})} \cdot \frac{(1 - \beta)^2 (\bar{\theta} - \bar{\theta})^2 + 2\beta \theta (1 - \beta)(\bar{\theta} - \bar{\theta})}{2\beta^2 (\theta - \bar{\theta})}
\]
\[
= \frac{\bar{\theta} + \theta}{2} - \frac{\theta - \theta - 2\beta(\bar{\theta} - 2\bar{\theta}) + \beta^2 (\bar{\theta} - 3\theta)}{2\beta^2}
\]
\[
= \frac{\bar{\theta} - 2\theta}{\beta} - \frac{\theta - \bar{\theta}}{2\beta^2} + 2\theta
\]

Expected per period voter utility contracting on \( x \) rule =
\[
\int^\beta \frac{\beta \theta}{\theta - \bar{\theta}} d(\theta) = \frac{\beta(\bar{\theta}^2 - \theta^2)}{2(\theta - \bar{\theta})} = \frac{\beta(\bar{\theta} + \theta)}{2}
\]

threshold rule > contracting on \( x \) iff
\[
\frac{\bar{\theta} - 2\theta}{\beta} - \frac{\theta - \theta}{2\beta^2} + 2\theta \geq \frac{\beta(\bar{\theta} + \theta)}{2}
\]
\[
2\beta \bar{\theta} - 4\beta \bar{\theta} - \bar{\theta} + \theta + 4\beta^2 \theta \geq \beta^3 (\bar{\theta} + \theta)
\]
\[
-(\bar{\theta} + \theta)\beta^3 + 4\theta \beta^2 + (2\bar{\theta} - 4\bar{\theta})\beta - (\bar{\theta} - \theta) \geq 0
\]
\[
(\beta - 1)(-\bar{\theta} + \theta - 3\theta)\beta + (\bar{\theta} - \theta) \geq 0
\]
\[
(\beta - 1)(-\bar{\theta} - \theta)^* \]
\[(\beta - \frac{(\bar{\theta} - 3\theta)}{2(\theta + \bar{\theta})} + \sqrt{(\bar{\theta} - 3\theta)^2 + 4(\theta^2 - \bar{\theta}^2)}) (\beta - \frac{(\bar{\theta} - 3\theta) - \sqrt{(\bar{\theta} - 3\theta)^2 + 4(\bar{\theta}^2 - \theta^2)}}{2(\theta + \bar{\theta})}) \geq 0\]

when \(1 \geq \beta \geq \frac{(\bar{\theta} - 3\theta) - \sqrt{(\bar{\theta} - 3\theta)^2 + 4(\bar{\theta}^2 - \theta^2)}}{2(\theta + \bar{\theta})}\)

This is not a very strict condition, as this implies if \(\theta \sim U[0,1]\), the threshold rule beats contracting on \(x\) if \(\beta \geq \frac{-1 + \sqrt{5}}{2} \approx 0.618\)

D) Comparing \(v_{t+1}\) from B) and C), we see that it is not clear when he earns the greater returns. Traditionally, we would expect the politician to earn more when information is asymmetric than when there is complete information. When voters can observe \(\theta\) they can usually hold the politician closer to his IR constraint. When \(\theta\) is private information, the politician gains from the threat that he will take everything from the voters in the low case.

However, here in the perfect information case, the voters give the politician large returns in low periods and nothing in high periods. Where the citizens are clearly made better off using the threshold strategy, that does not necessarily mean the politician will be made worse off, as their consumptions are not linear combinations of one another. Peculiar to the politician’s utility function and the threshold voter strategy in this problem, there may be distributions of \(U(\bar{\theta}, \theta)\) where perfect information actually makes both the politician and the voters better off.

3) These answers are only an outline. More complete answers would include paper citations.

A) Reasoning:

Ethnic fragmentation may lead to a society polarized into ethnic groups which compete with one another. As education is a public good, a single ethnic group would not internalize the full return to the investment, and hence under invest. Further, provision of the public good may erode the base of power for the governing group, and hence public good provision may not be efficient.

Criticism:
- Tiebout sorting. It may be that cities with less ethnic diversity also have different tastes for government services than more diverse cities.
- Ethnic fragmentation may be correlated with average income in cities.
- Reverse Causality.

Possible Solution:
- Find an instrument for ethnic diversity, perhaps distance to port or border.
- Control for average income in the city.

B) Reasoning:

Political institutions affect economic outcomes. Specifically, a country’s legal code will affect corruption in that country.
Criticism:
- "Multi-colinearity" legal institutions are highly correlated with political systems, societal mores and work ethic. Colonial origin can exert an effect upon a country through channels other than legal code.
- Colonialization strategy is not random.

Possible Solution:
- Instrument for colonialization with settler mortality.
- Attempt to control for societal mores and other mechanisms through which colonial origin would affect corruption other than through the legal environment.

C) See B)

D) Reasoning:
Theories for democracy affect on inequality are ambiguous. Democracy tends to be correlated with better economic systems, and rule by the median (income) voter. This median voter might vote to increase government redistribution to decrease inequality and increase the income of the median voter.

Criticism:
- Omitted variables, units of observation will be countries, which differ from one another in a variety of ways other than just through an index of democracy.
- It is unlikely that the relationship between democracy and inequality will be linear.
- Reverse Causality.

Possible Solution:
- Use a nonlinear specification
- Use a panel dataset.
- IV for inequality, say through natural resources in the country, financial crises, etc.

E) Reasoning:
Political instability in a country’s neighbors may cause increased uncertainty in investment decisions within a country. If returns from investment (or property rights in general) are decreased because of political uncertainty, economic performance should be reduced. Controlling for a country’s own instability is essential, as instability is likely to be regionally correlated.

Criticism:
- Omitted variables.
- Averaging all neighbors’ instability may not capture the true effect of instability.
- Possible spurious relationship if much of the variation is coming through countries in Africa, with low growth prospects and high levels of instability.
- Reverse Causality.

Possible Solution:
- Add controls for neighbors’ income.
- Try many specifications for neighbors’ political instability.
- Use panel data and looks for leads and lags in the correlation between inequality and instability.
F) Reasoning:
It is likely that inequality affects growth in many ways; political instability, class conflict, credit constraints, etc.

Criticism:
- Cross section ignores country fixed effects.
- Measurement error. Changes in five-year period likely all noise.
- Unclear whether inequality affects growth in the short run (5 years).
- Omitted variables.
- Possible non-linear relationship.
- Reverse Causality.

Possible Solution:
- Go with the cross section, but find an instrument for initial inequality.
- Use the panel over a longer period of time, and again instrument for inequality.

4) Answers will vary.

5) A) Workers have two options
Expropriate today: \( \mu f(k) + \beta(0) \)

Cooperate forever: \( \frac{\delta f(k)}{1 - \beta} \)

They will cooperate only if
\[
\frac{\delta f(k)}{1 - \beta} \geq \mu f(k)
\]

\[
\delta \geq \delta^* = \mu(1 - \beta)
\]

Capitalists will invest until the marginal net return on their investment equals their marginal return on the storage technology.

\[
(1 - \delta)f'(k) = 1
\]

\[
f'(k) = \frac{1}{1 - \delta}
\]

\[
k^* = f'^{-1}\left(\frac{1}{1 - \delta}\right)
\]
Hence $\delta \geq \mu(1 - \beta)$ and $k^* = f^{-1}\left(\frac{1}{1 - \delta}\right)$ are sustainable with the following strategy:

Workers: Expropriate $\delta f(k)$ in every period as long as there has never been any expropriation in the past, and $k > 0$ in all previous periods. If not, expropriate $\mu f(k)$ today and every period in the future.

Capitalists: Invest $k^*$ if workers have expropriated $\delta f(k)$ in every past period. Otherwise, invest 0.

B) Now, this übertcapitalist “frees” the capitalists from each setting their net marginal return equal to their storage technology. As long as each capitalist is doing better by investing the proscribed level of $k$ and $\delta$, she will invest. While the original set of $\delta$ and $k$ remain possible, new combinations also become sustainable. Worker’s constraints remain unchanged.

Capitalists now face the same choice as the workers did

Deviate today: $\max_k (1 - \mu)f(k) + (e - k) + \beta e \frac{1}{1 - \beta}$

Cooperate forever: $\frac{(1 - \delta)f(k) + (e - k)}{1 - \beta}$

From the first order condition, if a capitalist chooses to deviate, she will choose $k$ such that $(1 - \mu)f'(\hat{k}) = 1$. So capitalists will choose to cooperate as long as

$$\frac{(1 - \delta)f(k) + (e - k)}{1 - \beta} \geq (1 - \mu)f(\hat{k}) + (e - \hat{k}) + \beta e \frac{1}{1 - \beta}$$

$$(1 - \delta)f(k) - k \geq (1 - \beta)[(1 - \mu)f(\hat{k}) - \hat{k}]$$

There are a variety of $\delta$ which fulfill this inequality for any given level of $k$. Further, cooperation can support net marginal returns at $k$ less than the return from the storage technology, i.e. $(1 - \delta)f'(k) < 1$. While each capitalist is not maximizing by investing in the region $k > k^*$, the advantage in continuing to cooperate outweigh the loss.

The set of supportable $\delta$ and $k$ is larger in $k$ because capitalists do not force themselves to $(1 - \delta)f'(k) = 1$. While net marginal returns greater than one ($(1 - \delta)f'(k) > 1$) are not efficient, they are sustainable if a central capitalists dictates a single level of $\delta$ and $k$. The übertcapitalist
prevents each capitalists from individually maximizing, and instead only faces the individual rationality constraint of the capitalists. This permits \( \delta \) and \( k \) combinations that capitalists would individually wish to deviate away from. This is particularly apparent for \( k \) such that the net marginal return at \( k \) is less than the storage technology \((1-\delta)f'(k) < 1\).

C) If workers can increase \( \mu \), in part A) that changes both the upper and lower support of the sustainable set. The upper bound increase one for one with the increase in \( \mu \), whereas the lower bound increases by only \((1-\beta)\) for each one unit increase in \( \mu \). Capitalists will continue to set net marginal return equal to the storage technology. The increase in \( \mu \) will increase the size of the sustainable set. Hence for part A):

For part B) the changes to the support of the sustainable set are similar to part A). In addition, the increase in \( \mu \) also changes the capitalists’ cooperation constraint, making it less restrictive.

\[
(1 - \delta)f(k) - k \geq (1 - \beta) [(1 - \mu)f(\hat{k}) - \hat{k}]
\]

This amounts to more \( \delta \) and \( k \) combinations fulfilling the constraint and hence an even larger increase in the sustainable set than in A). Visually:

In neither case is it clear that the increase in \( \mu \) can lead to a Pareto improvement for both the capitalists and workers. While in case B), the change in \( \mu \) may lead to a change in \( \delta \) and \( k \), it is unclear if this change will benefit either or both the capitalist and worker, although it is possible. In case A), the increase in \( \mu \) would improve the returns of the workers, but could only lead to lower returns for the capitalists.
D) If workers and capitalist become less patient, the results are similar to C) above. In case A) the decrease in \( \beta \) raises the lower support of the sustainable set, while leaving the upper support unchanged.

\[
\delta \\
\mu \\
\frac{\mu(1-\beta')}{\mu(1-\beta)} \\
(1-\delta)f'(k) = 1
\]

For case B), the decrease in \( \beta \) again raises the lower support of the sustainable set, but also affects the capitalists' decision.

\[
(1-\delta)f(k) - k \geq (1-\beta) [(1-\mu)f(\hat{k}) - \hat{k}]
\]

By making the capitalists less willing to sacrifice consumption today for consumption tomorrow, this will decrease the sustainable set of \( \delta \) and \( k \) combinations.

Again, here the shrinking if of the sustainable set does not lead to an unambiguous improvement for each group. Hence the workers and capitalists becoming less patient does not necessarily lead to a Pareto improvement.