14.773 Political Economy of Institutions and Development
Problem Set 3
Due April 1, 2003

1) Grossman and Helpman Lobbying Model

Take an economy with three groups, rich, middle class and poor. A fraction \( \lambda \) of the agents are rich with income \( h^r \), \( \mu \) are middle class with income \( h^m \) and the rest are poor with income \( h^p \). There is a linear tax rate \( \tau \) imposed on all incomes with the proceeds distributed lump sum to all agents. Externalities associated with the tax scheme are associated with the tax rate and average income level through \( v(\tau)h \). Let \( v'(\tau) < 0 \), such that the externality is a dead weight loss from taxation.

Groups may attempt to lobby the government to influence its choice of tax rate. The government has a utility function \( g(\tau) = \sum_{i=1}^{3} C_i(\tau) + a \sum_{i=1}^{3} W_i(\tau) \). The utility function of each individual is \( W_j(\tau) = (1-\tau)h^j + [\tau - v(\tau)]h - C_j(\tau) \), \( j \in \{r, m, p\} \). Suppose the rich and middle class groups are organized, but the poor are not and can not make contributions to influence policy.

What is the equilibrium tax rate \( \tau \) chosen by the government? What is the contribution function for the rich and middle class groups? (You may assume it is a globally truthful contribution schedule as defined on p 840.)

Now let there be a small positive externality associated with the tax scheme, such that \( v'(\tau) > 0 \). Now what is the equilibrium tax rate \( \tau \) chosen by the government? What is the contribution function for the rich and middle class groups? (again assume global truthfulness)

2) Downsian Party Competition with Three Politicians.

Suppose there are three parties that first announce a policy platform \([0,1]\) and commit to it and a set of voters that vote for one of the three parties. Assume that all voters have single-peaked policy preferences, and these policy preferences are uniformly distributed over the unit interval. In the case of one or more parties selecting the same policy, voters randomize in the choice of which party for which they vote.

A) If the utility function of each party is only its probability of coming to power, what will each of the three parties announce as their policy platform?

B) If instead each party attempts to maximize the share of the vote it receives, what will each of the three parties announce as their policy platform? (hint, each party randomizes over a continuous support)

C) Give an example motivating why a political party might have either of these two forms of their utility function.

3) Median Voter Theorem with Closed Agenda

Let there be \( n \) possible discrete policies denoted \( q_1 \) to \( q_n \), and \( m > n \) voters with single peaked preferences. Define the policy preference of the median voter \( q_m \). Let voters vote in a closed agenda election including all policy choices in a prespecified order of policy versus policy contests, with the winning policy moving forward to compete with another policy in the next round.
A) Assume all actors vote sincerely. Show that this voting scheme will result in the policy preference of the median voter.

B) Assume that only one voter uses strategic voting, and all other voters vote sincerely. Will this voting scheme result in the policy preference of the median voter? Illustrate your answer with an example.

C) Now assume it is common knowledge that all voters may vote strategically, will this voting scheme result in the policy preference of the median voter? Give a sketch of a proof.

4) Lindbeck and Weibull Probabilistic Voting

A society is a two party democracy, with political parties R and D competing to maximize their vote share. Parties compete by proposing a tax rate $\tau$ with proceeds distributed lump sum to each member of society (ala Roberts’ or Meltzer and Richards’ model). Taxing income introduces some distortions, so while the government collects $\tau y$, only $[\tau - w(\tau)] \bar{y}$ is redistributed to each person, with $w'(\tau) > 0$, $w''(\tau) > 0$.

The society is stratified into $n$ groups. The size of each group various, but members of the same group have the same income, although differing political ideology. Let $y_{ij}$ denote the income of each individual in group $j$, $\sigma_{ij}$ the political leaning towards party $R$ of individual $i$ in group $j$ and $\alpha_{ij}$ the group’s size. Normalize $\sum_{k=1}^{n} \alpha_{k} = 1$, and let $\sigma_{ij} \sim U[\frac{-1}{2\phi}, \frac{1}{2\phi}]$.

Assume that individuals of the society all share a common utility function $U_{ij}(y, \sigma_{ij}) = c_{i} + [\sigma_{ij} + \delta] * I_{R}$

Where $I_{R}$ is the indicator function for $R$ coming to power and $\delta$ measures the average relative popularity of $R$ in the population as a whole, $\delta \sim U[\frac{-1}{2\psi}, \frac{1}{2\psi}]$.

A) Ignore the political leanings of each group and the relative popularity measure. If income is right skewed such that $\bar{y} > y_{median}$, what is the tax rate $\tau$ announced by each party?

B) Now assess the effects of ideological heterogeneity (that is, $\sigma_{ij}$ and $\delta$). Assume that neither party knows the realization of $\delta$ prior to announcing their tax rate and therefore perceives its vote share as a random event. What will be the tax schedules announced by each party? How does this policy differ from the one that would be enacted through the median voter theorem?

C) Now assume the parties can offer both a lump sum redistribution to all members of the society, and a group specific transfers $\omega_{ij} \geq 0$ to each group. The government budget constraint thus becomes $\tau \bar{y} = T + \sum_{k=1}^{n} \alpha_{k} + w(\tau) \bar{y}$, where $T$ is the lump sum transfer, the second terms represents the total cost of group specific transfers and the final term comes from the inefficiencies associated with the tax rate. Show that there is no pure strategy equilibrium for the game when $\delta$ is known in advance to be 0. Characterize
the equilibrium when \( \delta \) is random, and parties perceive their vote share to be probabilistic.

5) Roberts-Meltzer-Richard Model with Physical and Human Capital

Consider the following one-period economy populated by a mass 1 of agents. A fraction \( \lambda \) of these agents are capitalists, each owning capital \( k \). The remainder have only human capital, with human capital distribution \( F(h) \). Output is produced in competitive markets, with aggregate production function

\[
Y = K^{1-\alpha}H^\alpha
\]

Denote the market clearing rental price of capital by \( r \) and that of human capital by \( \omega \).

A) Suppose that agents vote over linear income tax, \( t \). Because of tax distortions, total tax revenue is

\[
\text{Tax} = \tau \left( \lambda rk + (1-\lambda)\omega \int hdF(h) \right) - \nu (\tau)
\]

where \( \nu \) is strictly increasing and convex, with \( \nu'(0) = 0 \) and \( \nu'(1) = \infty \) (why are these conditions useful?). Tax revenues are redistributed lump sum. Find the ideal tax rate for each agent. Find conditions under which preferences are single peaked, and determine the equilibrium tax rate. How does the equilibrium tax rate change when \( k \) increases? How does it change when \( \lambda \) increases? Explain.

B) Suppose now that agents vote over capital and labor income taxes, \( \tau_k \) and \( \tau_h \), with corresponding costs \( \nu(\tau_k) \) and \( \nu(\tau_h) \). Determine ideal tax rates for each agent. Suppose that \( \lambda < 1/2 \). Does a voting equilibrium exist? Explain. How does it change when \( \lambda \) increases? Explain why this would be different from the case with only one tax instrument?

C) In this model with two taxes, now suppose that agents first vote over the capital income tax, and then taking the capital income tax as given, they vote on the labor income tax. Does a voting equilibrium exist? Explain. If an equilibrium exists, how does the equilibrium tax rate change when \( k \) increases? How does this change when \( \lambda \) increases?