14.773 Political Economy of Institutions and Development  
Problem Set 5  
Due May 6, 2003

Question 1

Consider the following infinite horizon economy populated by two groups, denoted 1 and 2, of equal size. All agents in both groups maximize the expected present discounted value of income, with discount factor $\beta$. In any period one of the groups is in power while the other group is out of power. When either group is in power, it loses power with probability $q < 1/2$ in every period.

Income is generated in the following way: group $j$ has an asset stock of $A_{jt}$ at time $t$. Using these assets, it can produce income $A_{jt}(l_{jt})$ if it exerts labor $l_{jt}$ which costs $l_{jt}$ in terms of utility. Assume that income can be hidden in a non-taxable sector that generates a net return of $(1 - \mu)A_{jt}(l_{jt})$, for any assets not expropriated.

The net return to the group in the taxable sector is

$$(1 - e_{jt})(1 - T_{jt})A_{jt}(l_{jt}) - l_{jt}$$

where $T_{jt}$ is a tax rate faced by this group, and $e_{jt} \in [0, 1]$ denotes the proportion of group $j$’s assets that are expropriated in period $t$. $T_{jt}$ is group specific, and the income gained through taxation is costlessly transferred to the party in power.

The law of motion of assets, as a function of expropriation of assets, is given by:

$$A_{1t} = A_{1t-1} - e_{1t}A_{1t-1} + e_{2t}A_{2t-1}$$

$$A_{2t} = A_{2t-1} - e_{2t}A_{2t-1} + e_{1t}A_{1t-1}$$

where, recall that, $e_{jt}$ denotes whether group $j$ expropriates the other groups’ assets.

1. First suppose that asset expropriation is not allowed, so $e_{jt} = 0$, so the only decision each group takes is the tax rate it sets when in power. Characterize the pure strategy Markov Perfect Equilibria (MPE) of this repeated game. Show that the output level is less than first-best, and is constant over time.

2. Next suppose that the group in power can expropriate the assets of the other group (so the two decisions now are taxes and expropriation). Characterize the MPE, and show that output can actually be higher in this economy than the economy without asset expropriation. Explain why. Show also that now output is no longer constant, but fluctuates over time.

3. Next consider a model endogenizing $g$. In particular, imagine that the group out of power can choose to take power in any period but to do so must pay a non-pecuniary cost $c$. This cost $c$ is drawn each period from the distribution $G(c)$. First consider the case without asset expropriation. Show that there will exist a level of $c^*$ such that when $c \leq c^*$, the group out of power will take power (Hint: write the Bellman equations in terms of $c^*$, and obtain a fixed-point recursion for $c^*$).

4. Next consider the case with asset expropriation (where the group that comes to power can cost is the expropriate all the assets of the other group). Show that there will exist a level of $c^{**}$ such that when $c \leq c^{**}$, the group out of power will take power, and show that $c^{**} > c^*$. Show also that this economy with endogenous power switches has higher volatility than the corresponding economy with exogenous power switches.

5. Discuss whether two theoretical channels, highlighted by the model, linking security of property rights to economic instability are plausible. Feel free to give real world examples.
**Question 2**

Consider an economy populated by $\lambda$ rich agents who initially hold power, and $1 - \lambda$ poor agents who are excluded from power, with $\lambda < 1/2$. All agents are infinitely lived and discount the future at the rate $\beta$. Rich agents have income $\theta > 1$ and poor agents have income normalized to 1. In any given period, the decisive voter chooses the tax rate. Each rich agent can individually take his money to Miami, and in the process he loses a fraction $\phi$ of his income. The poor can undertake a revolution, and if they do so, it all future periods, they take a fraction $\mu_i$ of income of the rich (and the poor also keep their own income, and assume that the rich get 0 from then on). At the beginning of every period, the rich can also decide to extend the franchise to the poor, and this is irreversible.

1. First suppose that $\mu_i = \mu^l < \phi$. Find the Markov perfect equilibrium (MPE) of this game.
2. What happens if $\mu^l > \phi$?
3. Now suppose that $\mu_i = \mu^l$ with probability $1 - q$, and $\mu_i = \mu^h > \mu^l$ with probability $q$. Show that there exists an MPE where the rich extend the franchise, and from there on, a poor agent sets that tax rate. Explain why extension of the franchise is useful for rich agents? Why couldn’t the rich keep power and promise to set the same tax rate as the poor median voter? Is the assumption that the decision to extend franchise before the revolution decision important?
4. Does an increase in $\theta$ make extension of the franchise more or less likely? Why? Is this a good prediction? How would you change this implication? Quickly outline a model with a repression technology which would do the trick.
5. Now imagine that “international capital flows” improve, thus $\phi$ falls. Does this make democratization more or less likely? Does it make democratization more or less likely when the rich have a repression technology?

**Question 3**

Consider the following infinite horizon economy. Time is discrete and indexed by $t$. There is a set of citizens, with mass normalized to 1 and a ruler. All agents discount the future with the discount factor $\beta$, and have the utility function

$$u_t = \sum_{j=t}^{\infty} \beta^j [c_{t+j} - e_{t+j}],$$

where $c_{t+j}$ is consumption and $e_{t+j}$ is investment, and assume that the ruler undertakes no investment.

Each citizen $i$ has access to the following Cobb-Douglas production technology:

$$y_t^i = \frac{1}{1 - \alpha} A_t (e_t^i)^{1-\alpha},$$

where $A_t$ denotes state of technology and infrastructure at time $t$, which will be determined by the ruler. In addition, the ruler sets a tax rate $\tau_t$ on income. Also, each citizen can decide to hide a fraction $z_t^i$ of his output, which is not taxable, but hiding output is costly, so a fraction $\delta$ of it is lost in the process. So consumption of agent $i$ is:

$$c_t^i \leq ((1 - \tau_t)(1 - z_t^i) + (1 - \delta)z_t^i)y_t^i,$$

and tax revenues are
\[ T_t = \tau_t \int (1 - z_t^t)y_t^i di. \]

The ruler at time \( t \) decides how much to spend on \( A_{t+1} \) with technology:

\[ A_{t+1} = \left[ \frac{(1 - \alpha)q}{\alpha} G_t \right]^{1/\phi} \]

where \( G_t \) denotes government spending on infrastructure, and \( \phi < 1 \). This implies that the consumption of the ruler is

\[ c_t^R = T_t - G_t. \]

The timing of events within every period is as follows:

- The economy inherits \( A_t \) from government spending at time \( t - 1 \).
- Citizens choose investment, \( \{e_t^i\} \).
- The ruler sets tax rate \( \tau_t \).
- Citizens decide how much of their output to hide, \( \{z_t^i\} \).
- The ruler decides how much spent on next period’s infrastructure \( G_t \).

1. Find the first-best allocation in this economy.
2. Define a Markov Perfect Equilibrium (MPE) where strategies at time \( t \) depend on the payoff-relevant state of the game at time \( t \). Be specific about what the strategies are and what the state is.
3. Show that in a MPE \( \tau_t = \delta \) for all \( t \). Interpret what \( \delta \) corresponds to. What would you say is the difference between a society with high \( \delta \) and one with low \( \delta \)?
4. Given this, find the equilibrium level of investment by citizens and tax revenues as functions of \( A_t, T(A_t) \).
5. Explain why we could write the value function of the ruler as

\[ V(A_t) = \max_{A_{t+1}} \left\{ T(A_t) - \frac{\alpha}{\phi(1 - \alpha)} A_t^{\phi} A_{t+1}^{\phi} + \beta V(A_{t+1}) \right\}, \]

and using this, find the equilibrium spending on infrastructure, \( G_t \).
6. What is the effect of \( \delta \) on spending on infrastructure and equilibrium investments. Does a decline in \( \delta \) always imply greater investments? If not, why not? Calculate the equilibrium level output and find the output maximizing level of \( \delta \). Discuss what this means, and how you could map this to reality.
7. Briefly discuss how the non-Markovian equilibria of this game would look like. If you can, set up the problem, and conjecture what type of solutions there might exist (you do not have to compute the solution sets).