Lecture 5 - PN Junction and MOS
Electrostatics (II)

PN Junction in Thermal Equilibrium

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Reading assignment:

Howe and Sodini, Ch. 3, §§3.3-3.6
Key questions

• What happens if the doping distribution in a semiconductor abruptly changes from n-type to p-type?

• Is there a simple description of the electrostatics of a pn junction?
1. Introduction to pn junction

- pn junction: p-region and n-region in intimate contact

- Why is the p-n junction worth studying?

It is present in virtually every semiconductor device!

Example: CMOS cross section

Understanding p-n junction is essential to understanding transistor operation.
2. Electrostatics of p-n junction in equilibrium

Focus on intrinsic region:

Doping distribution of abrupt p-n junction:
What is the carrier concentration distribution in thermal equilibrium?

First think of two sides separately:

Now bring them together. What happens?

Diffusion of electrons and holes from majority carrier side to minority carrier side until drift balances diffusion.
Resulting carrier profile in thermal equilibrium:

- Far away from metallurgical junction: nothing happens
  - two quasi-neutral regions
- Around metallurgical junction: carrier diffusion must cancel drift
  - space-charge region
In a linear scale:

Thermal equilibrium: balance between drift and diffusion

Can divide semiconductor in three regions:

- two quasi-neutral n- and p-regions (QNR’s)
- one space charge region (SCR)

Now, want to know $n_o(x), p_o(x), \rho(x), E(x)$, and $\phi(x)$. Solve electrostatics using simple, powerful approximation.
3. The depletion approximation

- Assume QNR’s perfectly charge neutral
- Assume SCR depleted of carriers (depletion region)
- Transition between SCR and QNR’s sharp
  (must calculate where to place $-x_{po}$ and $x_{no}$)

\[
\begin{align*}
  x &< -x_{po} \quad p_o(x) = N_a, \quad n_o(x) = \frac{n_i^2}{N_a} \\
  -x_{po} < x < 0 \quad p_o(x), \quad n_o(x) &\ll N_a \\
  0 < x < x_{no} \quad n_o(x), \quad p_o(x) &\ll N_d \\
  x_{no} < x \quad n_o(x) = N_d, \quad p_o(x) = \frac{n_i^2}{N_d}
\end{align*}
\]
**Space Charge Density**

\[
\begin{align*}
\rho(x) &= 0 & x < -x_{po} \\
&= -qN_a & -x_{po} < x < 0 \\
&= qN_d & 0 < x < x_{no} \\
&= 0 & x_{no} < x
\end{align*}
\]
• **Electric Field**

Integrate Gauss’ equation:

\[ E(x_1) - E(x_2) = \frac{1}{\epsilon} \int_{x_1}^{x_2} \rho(x) \, dx \]

\[ E(x) = \begin{cases} 
0 & \text{if } x < -x_{po} \\
\frac{-qN_a}{\epsilon} x \bigg|_{-x_{po}}^{x} & \text{if } -x_{po} < x < 0 \\
\frac{qN_d}{\epsilon} (x - x_{no}) & \text{if } 0 < x < x_{no} \\
0 & \text{if } x_{no} < x
\end{cases} \]
- **Electrostatic Potential**
  (with $\phi = 0 \ @ \ n_o = p_o = n_i$):

\[
\phi = \frac{kT}{q} \ln \frac{n_o}{n_i} \quad \phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}
\]

In QNR's, $n_o$, $p_o$ known $\Rightarrow$ can determine $\phi$:

- in p-QNR: $p_o = N_a \Rightarrow \phi_p = -\frac{kT}{q} \ln \frac{N_a}{n_i}
- in n-QNR: n_o = N_d \Rightarrow \phi_n = \frac{kT}{q} \ln \frac{N_d}{n_i}

Built-in potential:

\[
\phi_B = \phi_n - \phi_p = -\frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}
\]

General expression: did not use depletion approximation.
To get $\phi(x)$ in between, integrate $E(x)$:

$$\phi(x_1) - \phi(x_2) = -\int_{x_1}^{x_2} E(x) \, dx$$
- $x < -x_{po}$ \quad $\phi(x) = \phi_p$

- $-x_{po} < x < 0$ \quad $\phi(x) - \phi(-x_p) = -\int_{-x_{po}}^{x} -\frac{qN_a}{\epsilon}(x + x_{po}) \, dx$
  \[= \frac{qN_a}{2\epsilon}(x + x_{po})^2\]
  \[\phi(x) = \phi_p + \frac{qN_a}{2\epsilon}(x + x_{po})^2\]

- $0 < x < x_{no}$ \quad $\phi(x) = \phi_n - \frac{qN_d}{2\epsilon}(x - x_{no})^2$

- $x_{no} < x$ \quad $\phi(x) = \phi_n$

Almost done...
Still don’t know \( x_{no} \) and \( x_{po} \) ⇒ need two more equations

1. Require overall charge neutrality:

\[
q N_a x_{po} = q N_d x_{no}
\]

2. Require \( \phi(x) \) continuous at \( x = 0 \):

\[
\phi_p + \frac{q N_a}{2\epsilon} x_{po}^2 = \phi_n - \frac{q N_d}{2\epsilon} x_{no}^2
\]

Two equations with two unknowns. Solution:

\[
x_{no} = \sqrt{\frac{2\epsilon\phi_B N_a}{q(N_a + N_d) N_d}} \\
x_{po} = \sqrt{\frac{2\epsilon\phi_B N_d}{q(N_a + N_d) N_a}}
\]

Now problem completely solved.
Other results:

Total width of space charge region:

\[ x_{do} = x_{no} + x_{po} = \sqrt{\frac{2\varepsilon\phi_B (N_a + N_d)}{qN_aN_d}} \]

Field at metallurgical junction:

\[ |E_o| = \sqrt{\frac{2q\phi_B N_aN_d}{\varepsilon(N_a + N_d)}} \]
Three cases:

- **Symmetric junction:** \( N_a = N_d \Rightarrow x_{po} = x_{no} \)
- **Asymmetric junction:** \( N_a > N_d \Rightarrow x_{po} < x_{no} \)
- **Strongly asymmetric junction:**
  - *i.e.* \( p^+n \) junction: \( N_a \gg N_d \)

\[
x_{po} \ll x_{no} \simeq x_{do} \simeq \sqrt{\frac{2\varepsilon \phi_B}{qN_d}} \propto \frac{1}{\sqrt{N_d}}
\]

\[
|E_o| \simeq \sqrt{\frac{2q\phi_B N_d}{\epsilon}} \propto \sqrt{N_d}
\]

The lowly-doped side controls the electrostatics of the \( pn \) junction.
4. Contact potentials

Potential distribution in thermal equilibrium so far:

Question 1: \textit{If I apply a voltmeter across diode, do I measure $\phi_B$?}

- yes
- no
- it depends

Question 2: \textit{If I short diode terminals, does current flow on outside circuit?}

- yes
- no
- sometimes
We are missing *contact potential* at metal-semiconductor contacts:

Metal-semiconductor contacts: junctions of dissimilar materials
⇒ built-in potentials: $\phi_{mn}$, $\phi_{mp}$

Potential difference across structure must be zero
⇒ cannot measure $\phi_B$!

\[ \phi_B = \phi_{mn} + \phi_{mp} \]
Key conclusions

- Electrostatics of pn junction in equilibrium:
  – a *space-charge region*
  – surrounded by two *quasi-neutral regions*
  ⇒ built-in potential across p-n junction

- To first order, carrier concentrations in space-charge region are much smaller than doping level
  ⇒ *depletion approximation.*

- Contact potential at metal-semiconductor junctions:
  ⇒ from contact to contact, there is no potential build-up across pn junction